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CSE 312: Foundations of Computing II
Section #7
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1. The Pareto distribution, was discovered by Vilfredo Patero and is used in a wide array of fields but particularly social sciences and economics. It is a density function with a slowly decaying tail, for example it can describe the wealth distribution (a small group at the top holds most of the wealth). The PDF is given by:

$$f_X(x; m, \alpha) = \frac{\alpha m^\alpha}{x^{\alpha+1}}$$

where $x \geq m$ and real $\alpha, m > 0$. m describes the minimum value that X takes on (scale) and α is the shape. So the range of X is $\Omega_X = [m, \infty)$. Assume that m is given and that $x_1, x_2, \ldots, x_n$ are i.i.d. samples from the Pareto distribution. Find the MLE estimation of α.

2. A weather forecaster predicts sun with probability θ_1, clouds with probability $\theta_2 - \theta_1$, rain with probability $\frac{1}{2}$ and snow with probability $\frac{1}{2} - \theta_2$. This year, there have been 55 sunny days, 100 cloudy days, 160 rainy days and 50 snowy days. What is the maximum likelihood estimator for θ_1 and θ_2?

3. Let $X_1, \ldots, X_n$ be a random sample from the distribution with PDF $f_X(x \mid \theta) = (\theta^2+\theta)x^{\theta-1}(1-x)$ for $0 < x < 1$ and $\theta > 0$. What is the MOM estimator for θ?

4. Let $x_1, x_2, \ldots, x_n$ be a random sample from a distribution with PDF $f(x; \theta, k) = \theta \frac{k^\theta}{x^{\theta+1}}$ where $x \geq k$ and k, θ are positive real numbers. The expectation is given by

$$E[X] = \frac{k\theta}{\theta - 1}$$

Determine

(a) the maximum likelihood estimator of k and θ

(b) the method of moments estimator of θ

(c) suppose the observed samples were **x** = (0.18, 0.94, 0.54, 0.11, 0.62, 0.45). What are the MLE and MOM estimators of k and θ to 3 decimal places? (Use the same value of k from the MLE estimator in (a) for both of your calculations.)

5. Suppose $x_1, \ldots, x_{2n}$ are iid realizations from the Laplace density (double exponential density)

$$f_X(x \mid \theta) = \frac{1}{2} e^{-|x-\theta|}$$

Find the MLE for θ. For this problem, you need not verify that the MLE is indeed a maximizer. You may find the **sign** function useful:

$$\text{sgn}(x) = \begin{cases} +1, & x > 0 \\ -1, & x < 0 \end{cases}$$

(in our case undefined at 0)