

Section #6

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1. Let X and Y be independent geometric random variables with the same success parameter $p < 1$. Find the distribution of $X + Y$, using n as the value of $X + Y$. That is solve for: $\mathbb{P}(X + Y = n)$ in terms of p .
2. A new diner specializing in waffles opens on our street. It will be open 24 hours a day, seven days a week. It is assumed that the inter-arrival times between customers will be i.i.d. Exponential random variables with mean 10 minutes. Approximate the probability that the 120th customer will arrive after the first 21 hours of operation.
3. You hold a stick of unit length (1). Someone comes along and breaks off a random piece at some point $Y \sim Unif(0, 1)$. Now you hold a stick of length Y . Another person comes along and breaks off another piece from the remaining part of the stick that you hold at point $X \sim Unif(0, Y)$. You are left with a stick of length X . Find the PDF f_X , mean $\mathbb{E}[X]$ and variance $Var(X)$.
4. An urn has 12 balls, 5 red ones and 7 green ones. Draw 3 balls. Let X denote the number of red balls in the sample. Compute $Var(X)$ when sampling is done:
 - (a) With replacement
 - (b) Without replacement
5. Now let's consider the general case. Suppose we have N balls in an urn, K red balls, $(N-K)$ green balls, and we draw n times. Denote the total number of red balls to be X . What is $Var(X)$?