CSE 312: Foundations of Computing II
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Lecture Topics: 2.3 Independence, 3.1 Discrete Random Variables Basics

[**Tags**: PSet1 Q10ac, Conditional Independence]

1. A website wants to detect if a visitor is a robot or a human. They give the visitor seven CAPTCHA tests that are hard for robots but easy for humans. If the visitor fails any of the tests, they are flagged as a robot. The probability that a human succeeds at a single test is 0.95, while a robot only succeeds with probability 0.3. Assume all tests are independent. The percentage of visitors on this website that are robots is 10%; all other visitors are human.
   a. If a visitor is actually a robot, what is the probability they get flagged (the probability they fail at least one test)?
   b. Compute the probability that a random visitor is flagged. (Helps with part (c)).

Solution: Watch lecture ☺.

[**Tags**: Independence, Random Variables, PMFs, Expectation, PSet2 Q8 (Similar)]

2. There are 3 people in Alex's family; his mom, dad, and sister. Each family member decides whether or not they want to come to lunch in his social-distancing home restaurant, independently of the others.
   - Mom wants to come with probability 0.8.
   - Dad wants to come with probability 0.6.
   - Sister wants to come with probability 0.1.

Unfortunately, if all 3 of them want to come, he must turn one of them away ☹ since the restaurant capacity is 2 guests. Otherwise, he will take everyone that comes. Let X be the number of customers that Alex serves at lunch.

   a. What is the range Ω_X, the PMF $p_X(k)$, and the expectation $E[X]$?
   b. If he charges everyone who comes \$10, but it costs him $\$50$ to make all the food, what is his expected profit?

Solution:

   a. The range is $\Omega_X = \{0,1,2\}$ since we can have anywhere from 0 to 2 people. By independence,

$$P(X=0) = P(M^C, D^C, S^C) = P(M^C)P(D^C)P(S^C) = 0.2 \cdot 0.4 \cdot 0.9 = 0.072$$

$$P(X=1) = P(M, D^C, S^C) + P(M^C, D, S^C) + P(M^C, D^C, S)$$
$$= 0.8 \cdot 0.4 \cdot 0.9 + 0.2 \cdot 0.6 \cdot 0.9 + 0.2 \cdot 0.4 \cdot 0.1 = 0.404$$

$$P(X=2) = 1 - P(X=0) - P(X=1) = 0.524$$

$$p_X(k) = \begin{cases} 0.072, & k=0 \\ 0.404, & k=1 \\ 0.524, & k=2 \end{cases}$$

$$E[X] = \sum_{k \in \Omega_X} k \cdot p_X(k) = 0 \cdot 0.072 + 1 \cdot 0.404 + 2 \cdot 0.524 = 1.452$$

b. The profit is $P = 10X - 50$, so $E[P] = E[10X - 50] = 10E[X] - 50 = 14.52 - 50 = -35.48$.

[**Tags**: Chain Rule, Inclusion-Exclusion]

3. Suppose n people sit around a table. Each person orders a different dish, but the waiter did not mark positions unfortunately. He has the correct n dishes, but gives a random dish to each person (each of the $n!$ assignments is equally likely). What is the probability that no one has the dish they ordered placed in front of them?

Solution: Go to recitation tomorrow!