1. A website wants to detect if a visitor is a robot or a human. They give the visitor seven CAPTCHA tests that are hard for robots but easy for humans. If the visitor fails any of the tests, they are flagged as a robot. The probability that a human succeeds at a single test is 0.95, while a robot only succeeds with probability 0.3. Assume all tests are independent. The percentage of visitors on this website that are robots is 10%; all other visitors are human.
   a. If a visitor is actually a robot, what is the probability they get flagged (the probability they fail at least one test)?
   b. Compute the probability that a random visitor is flagged. (Helps with part (c)).

2. There are 3 people in Alex’s family; his mom, dad, and sister. Each family member decides whether or not they want to come to lunch in his social-distancing home restaurant, independently of the others.
   - Mom wants to come with probability 0.8.
   - Dad wants to come with probability 0.6.
   - Sister wants to come with probability 0.1.

Unfortunately, if all 3 of them want to come, he must turn one of them away since the restaurant capacity is 2 guests. Otherwise, he will take everyone that comes. Let $X$ be the number of customers that Alex serves at lunch.
   a. What is the range $\Omega_X$, the PMF $p_X(k)$, and the expectation $E[X]$?
   b. If he charges everyone who comes $10, but it costs him $50 to make all the food, what is his expected profit?

3. Suppose $n$ people sit around a table. Each person orders a different dish, but the waiter did not mark positions unfortunately. He has the correct $n$ dishes, but gives a random dish to each person (each of the $n!$ assignments is equally likely). What is the probability that no one has the dish they ordered placed in front of them?