CSE 312: Foundations of Computing II
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Lecture Topics: 8.4 Introduction to Hypothesis Testing

[**Tags:** Hypothesis Testing]

1. Set up null and alternative hypotheses for the following scenarios. Some of the information is not relevant in setting these up.
   a. Alex believes the average salary of software engineers in the Bay Area is higher than that of Seattle. Let μ_x denote the true average salary of software engineers in the Bay Area and $\bar{x}$ the sample average of 312 samples, and μ_y denote the true average salary of software engineers in Seattle and $\bar{y}$ the sample average of 211 samples.
   b. Mitchell believes that the average temperature in Canada is different from -10 degrees Celsius. Let μ_x denote the true average temperature in Canada and $\bar{x}$ the sample average of 111 samples.
   c. Scott believes that there is less spread (variance) in height of corgis on CatIsland than DogIsland. Let σ_C^2 denote the true variance of corgi heights on CatIsland and s_C^2 the sample variance of 321 samples, and σ_D^2 denote the true variance of corgi heights on DogIsland and s_D^2 the sample average of 811 samples.

Solution:

These all relate to the TRUE parameter of interest and not the SAMPLE one (e.g., no $\bar{x}$ or any samples in the hypothesis). We use the samples to TEST the hypothesis later ☺.

a. $H_0: \mu_x = \mu_y$ and $H_A: \mu_x > \mu_y$ (one-sided)
b. $H_0: \mu_x = -10$ and $H_A: \mu_x \neq -10$ (two-sided)
c. $H_0: \sigma_C^2 = \sigma_D^2$ and $H_A: \sigma_C^2 < \sigma_D^2$ (one-sided)

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2. Alex thinks that the typical CSE student spends 125 hours in the "labs" per quarter working on homework. Pemi thinks he is incorrect, but isn't sure whether it is an overestimate or underestimate. After sampling 49 random CSE students, Pemi observes each of their true hours spent $x_1, \dots, x_{49}$ and finds the sample mean to be $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = 122$ (and sample variance of $s^2 = 12^2$). Help Pemi conduct a hypothesis test for his claim.

Solution:

We follow the procedure in the slides:

1. **Make a claim.**

The typical CSE student does NOT spend 125 hours per quarter working on homework.

2. **Set up the hypotheses.**

Let μ denote the true average amount of hours a CSE student spends per quarter working on homework in the labs. (that's a mouthful)

$$H_0 : \mu = 125 \qquad H_A : \mu \neq 125$$

3. **Choose a significance level.**

Let's say $\alpha = 0.05$.

4. **Collect data (already done).**

5. **Compute a p-value.**

Under the null hypothesis, for a single sample, $\mu = 125$ and $\sigma^2 = 12^2$ (we use the maximum likelihood estimate of σ^2 which is the sample variance s^2 given.)

By the CLT, $\bar{X} \approx N\left(125, \frac{12^2}{49}\right)$. The observed difference between the hypothesized mean and the sample mean is $|125 - 122| = 3$.

$$\begin{aligned} p = P(\text{data } \textbf{at least as} \text{ extreme}) &= P(|\bar{X} - \bar{x}| \geq 3) \\ &= P(\bar{X} \leq 122) + P(\bar{X} \geq 128) \\ &= P\left(Z \leq \frac{122 - 125}{\sqrt{\frac{12^2}{49}}}\right) + P\left(Z \geq \frac{128 - 125}{\sqrt{\frac{12^2}{49}}}\right) \\ &= P(Z \leq -1.75) + P(Z \geq 1.75) \\ &= 2P(Z \geq 1.75) \text{ [symmetry]} \\ &= 0.0801 \end{aligned}$$

6. **State your conclusion.**

Since our p-value of $0.0801 > \alpha = 0.05$, we fail to reject the null hypothesis. There is insufficient evidence to show that the true mean is any different from 125.

Note that if we had a one-sided hypothesis that $H_A : \mu < 125$ instead, we would have our p-value halved (only one of the two terms) to ≈ 0.04, and we would have rejected the null hypothesis!