

CSE 312: Foundations of Computing II

Instructor: Alex Tsun

Date: 08/05/20

Lecture Topics: 7.3 Method of Moments Estimation, 7.4 Beta/Dirichlet Distributions

[Tags: Estimation]

1. Suppose $\mathbf{x} = (x_1, \dots, x_n)$ are iid samples from the following distributions. Estimate the parameter(s) using your favorite technique (MLE or MoM). **Hint:** Use MoM.
 - a. The *Gamma*(r, λ) distribution. Estimate both r and λ .
 - b. The *Rayleigh*(σ) distribution with density $f_X(x; \sigma) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2}, x \geq 0$ with expectation $\sigma \sqrt{\frac{\pi}{2}}$.

[Tags: Beta/Dirichlet]

2. Suppose we roll a (possibly unfair) 4-sided die 29 times. Then, the number of times each digit appears is $\mathbf{X} = (X_1, X_2, X_3, X_4) \sim \text{Mult}_4(n = 29, \mathbf{p})$, where $\mathbf{p} = (p_1, p_2, p_3, p_4)$ is unknown. We happened to observe 5 ones, 7 twos, 6 threes, and 11 fours.
 - a. A *Beta*(α_1, β_1) rv would be suitable to model our belief on p_1 (the probability of rolling a one) with what parameters α_1, β_1 ?
 - b. A *Beta*(α_2, β_2) rv would be suitable to model our belief on p_2 (the probability of rolling a two) with what parameters α_2, β_2 ?
 - c. Let's instead say we wanted to jointly model all the unknown parameters \mathbf{p} . A *Dirichlet*($\boldsymbol{\gamma}$) would be suitable, more efficient than modelling all four separately, and also enforce that $\sum_{i=1}^4 p_i = 1$. Which parameter vector $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \gamma_3, \gamma_4)$ would best model our belief?