

[Tags: Multinomial, Multivariate Hypergeometric]

1. You work at a carnival and offer a simple game for children to play. The children get to choose a ticket from a bucket and based on the color of the ticket they will get different prizes. There are five colors of tickets: red, blue, green, and yellow which there are 24 of each, and gold which there are 4 of for a total of 100 tickets. Every day, 40 kids come to play the game. What is the probability that 9 of each of the red, blue, green, and yellow tickets and all 4 gold tickets are chosen on a given day if:
 - a. Your boss is present, so you have to make the game fair and replace each ticket after it is chosen?
 - b. Your boss isn't around, so you are lazy don't replace the tickets after each draw?

Solution:

- a. This is a multinomial distribution since we have replacement so we have:

$$\mathbf{X} \sim Mult_5 \left(n = 40, \mathbf{p} = \left(\frac{24}{100}, \frac{24}{100}, \frac{24}{100}, \frac{24}{100}, \frac{4}{100} \right) \right)$$

With X_1, X_2, X_3 and X_4 describing the number of red, blue, green, and yellow tickets pulled respectively and X_5 describing the number of gold tickets pulled.

We have $\mathbf{p} = \left(\frac{24}{100}, \frac{24}{100}, \frac{24}{100}, \frac{24}{100}, \frac{4}{100} \right)$ so p_1, p_2, p_3 and p_4 describe the probability of red, blue, green, and yellow tickets being pulled and p_5 describes the probability of gold tickets pulled.

Now, we want to solve for the probability of 9 of each of the red, blue, green, and yellow tickets and all 4 gold tickets being pulled and we have:

$$p_{X_1, X_2, X_3, X_4, X_5}(9, 9, 9, 9, 4) = \binom{40}{9, 9, 9, 9, 4} \left(\frac{24}{100} \right)^{9+9+9+9} \left(\frac{4}{100} \right)^4 = 0.000244$$

- b. Now, since we are without replacement, we have a multivariate hypergeometric and have:

$$\mathbf{X} \sim MVHG_5(N = 100, \mathbf{K} = (24, 24, 24, 24, 4), n = 40)$$

With X_1, X_2, X_3 and X_4 describing the number of red, blue, green, and yellow tickets pulled respectively and X_5 describing the number of gold tickets pulled.

We have $\mathbf{K} = (24, 24, 24, 24, 4)$ so k_1, k_2, k_3 and k_4 describe the initial number of red, blue, green, and yellow tickets and k_5 describes the initial number of gold tickets.

Now, we want to solve for the probability of 9 of each of the red, blue, green, and yellow tickets and all 4 gold tickets being pulled and we have:

$$p_{X_1, X_2, X_3, X_4, X_5}(9, 9, 9, 9, 4) = \frac{\binom{24}{9} \binom{24}{9} \binom{24}{9} \binom{24}{9} \binom{4}{4}}{\binom{100}{40}} = 0.000213$$