CSE 312: Foundations of Computing II
Instructor: Alex Tsun
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Lecture Topics: 5.6 Moment Generating Functions, 5.7 Limit Theorems

[**Tags:** MGFs]

1. We'll practice using MGFs.
   a. Let $X \sim Geo(p)$. Give a formula for $M_X(t)$, and specify for which values of t the formula converges. Recall the geometric series formula: $\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}$ holds for $|r| < 1$.
   b. Let $Y \sim NegBin(r, p)$. Give a formula for $M_Y(t)$.
   c. Set up formulas to compute $E[Y]$ and $Var(Y)$ using the MGF M_Y (but don't compute anything).

[**Tags:** CLT]

2. Use the CLT to approximate the following probabilities. Don't forget to apply the continuity correction (only if necessary).
   a. Suppose we roll a fair 10-sided die until we get 100 sevens. What is the probability it takes at least 1050 rolls until this happens?
   b. Let X be the sum of 10,000 real numbers, and Y be the same sum, but with each number rounded to the nearest integer before summing. If the fractions rounded off are independent and each one is uniformly distributed over $(-0.5, +0.5)$, use the Central Limit Theorem to estimate the probability that $|X - Y| > 50$. Noticing that $|X - Y|$ could have been as great as 5,000, look at your answer and think about what it says. (As a small example with sums of 4 real numbers, suppose that $X = 3.2 + 1.92 + (-3.6) + 5.7$. Then Y would be the sum of each of those terms when rounded to the nearest integer: $Y = 3 + 2 + (-4) + 6 = 7$. So, $|X - Y| = 1.3$. The fractions rounded off in this case are $(0.2, -0.08, 0.4, -0.3)$ and the assumption is that these fractions are independent and uniformly distributed in the real interval $(-0.5, +0.5)$.

[**Tags:** CLT, Law of Total Expectation]

3. Megha has a health condition that requires unpredictable amounts of medication. Every day, there is a 20% chance that she feels perfectly fine and requires no medicine. Otherwise, she needs to take a dose of medication. The necessary dose is equally likely to be any value in the continuous range 1 to 5 ounces. How much medicine she needs on any given day is independent of all other days. Megha's insurance will fully cover 90 ounces of medicine for each 30-day period. What is the probability that 90 ounces will be enough for the next 30 days? Make your life easier by using Central Limit Theorem.