CSE 312: Foundations of Computing II
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Lecture Topics: 4.3 The Normal RV, 4.4 Transforming Continuous RVs

[**Tags:** PSet3 Q4ab, The Normal RV]
1. Suppose the time that Java takes to sort a 1,000,000 length array is approximately
   $J \sim \mathcal{N}(\mu = 46, \sigma^2 = 6^2)$ milliseconds (ms), since it uses the (randomized) QuickSort Algorithm.
   a. Python initially implements a (deterministic) MergeSort Algorithm, and it always finishes in $P = 49$ ms. What is the probability that Java sorts a single 1,000,000 length array faster than Python does? Show your work and give your answer rounded to 4 decimal places.
   b. Python attempts to implement QuickSort as well, but did it less efficiently. Its runtime is approximately $P \sim \mathcal{N}(\mu = 55, \sigma^2 = 8^2)$. What is the probability that Java sorts a single 1,000,000 length array faster than Python does? Show your work and give your answer rounded to 4 decimal places.
   c. The remaining parts are left for you ☺.

[**Tags:** Transforming Continuous RVs]
2. Suppose $X \sim Exp\left(\lambda = \frac{1}{2}\right)$ is the waiting time in hours until your pizza delivery arrives, and suppose we decide to tip $Y = g(X) = \frac{24}{X+1}$ dollars.
   a. What is the range, PDF, and CDF of X? Hint: You can look this up.
   b. What is the range Ω_Y?
   c. Find $F_Y(y)$ using the CDF method, then find $f_Y(y)$ afterwards.
   d. Find $f_Y(y)$ using the explicit formula, after verifying the monotonicity and invertibility criteria.
   e. Set up integrals for $E[Y]$ in two ways: one with LOTUS and $f_X(x)$, and one with $f_Y(y)$. Explicitly define your limits of integration and the integrand so that one could enter your integral into WolframAlpha.

[Bonus!]
3. Suppose $X \sim Unif(-1,1)$ (continuous), then find the PDF of $Y = X^2$.