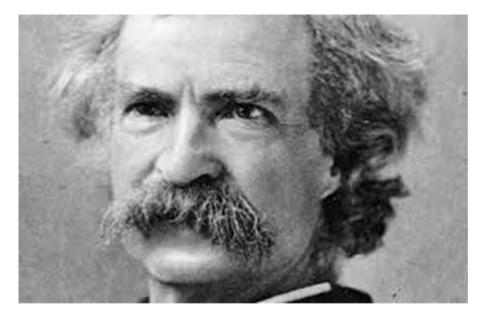
#### Random Quote

#### "There are three kinds of lies: lies, damned lies, and statistics."

- Mark Twain



# HODGEPODGE

#### ANNA KARLIN Most slides by Alex Tsun and by Maya Bar-Hillel

SOME SLIDES BASED OFF OF THE BOOK: "HOW TO LIE WITH STATISTICS" BY DARRELL HUFF AND THIS YOUTUBE VIDEO

#### BOTH GROUPS HAD THE SAME BRILLIANT IDEA

Larry Page and Sergey Brin (Ph.D. students at Stanford)

• Took the idea and founded Google, making billions

Jon Kleinberg (professor at Cornell)

 MacArthur Genius Prize, Nevanlina Prize and many other academic honors.

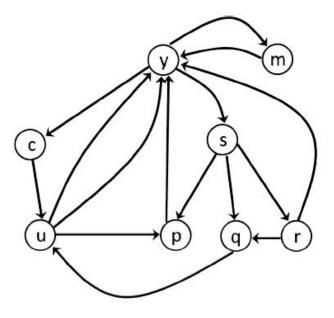






#### PAGERANK

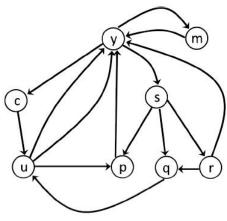
- Key idea is hyperlink analysis: take into account the directed graph structure of the web
- Hyperlink confers authority.
- Like a citation, a vote of quality



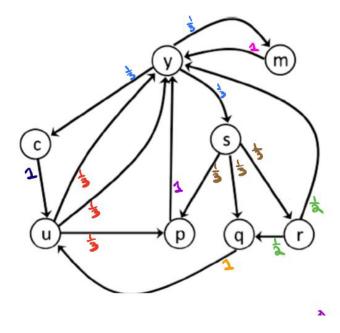
#### IDEA UNDERLYING PAGERANK



- Web page has **high quality** if it's linked to by lots of **high quality** pages.
- A page is high quality if it links to lots of high quality pages.
- If web page x has d outgoing links, one of which goes to y, then this contributes 1/d to the impor
- But, we want to take importance/quality o account.
- Recursive definition!



$$\begin{aligned}
Q_{P} &= quality & gage P \\
Q_{Y} &= q_{u} \cdot \frac{2}{3} + q_{P} \cdot 1 + q_{r} \cdot \frac{1}{2} + q_{m} \cdot 1 \\
Q_{P} &= q_{S} \cdot \frac{1}{3} + q_{u} \cdot 1
\end{aligned}$$



#### THESE ARE THE EQUATIONS qP = q

• Look familiar?

 $\pi P = \pi$ 



- Stationary distribution for what Markov chain?
- The Markov chain of a random surfer!!

#### SOME ISSUES

- Dangling nodes (dead ends)
- Rank sinks group of pages that only link to each other.

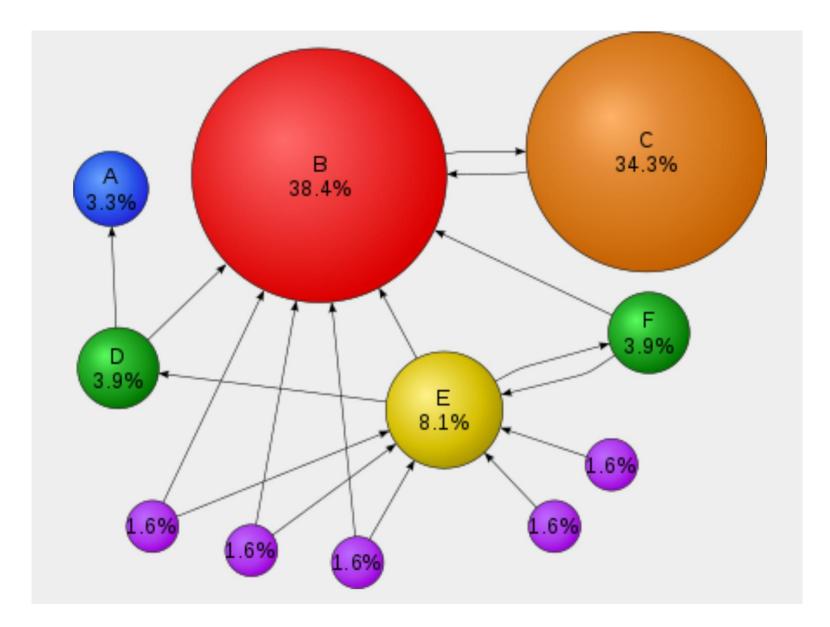
#### FINAL RANDOM SURFER MODEL



- Markov chain whose states are all the web pages in the world
- At each step, random surfer is looking at some web page. Next:
  - With probability p, follow a random link on that web page.
  - $\circ$  With probability 1-p, go to a uniformly random page on the web.

Compute stationary distribution for this Markov chain. Define Pagerank, i.e., ``quality" of a web page x to be its stationary probability  $\pi_x$ 

On a query, return pages with "good textual match". Rank them by their Pagerank.



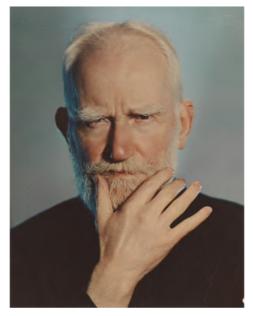
#### NOWADAYS

• Tons more secret sauce to ranking search results...

#### Random Quote

"It is the mark of a truly intelligent person to be moved by statistics"

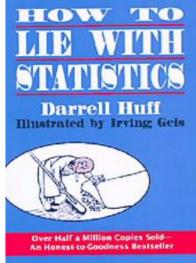
- George Bernard Shaw



## The Book



- Published in 1954, over 500,000 copies sold
- "A great introduction to the use of statistics, and a great refresher for anyone who's already well versed in it" - Bill Gates.



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## The Book



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- "A great introduction to the use of statistics, and a great refresher for anyone who's already well versed in it" Bill Gates.
- Doesn't teach how to lie with statistics, but how we are/can be lied to using statistics
- In the current age, we are lied to all the time, e.g., by **politicians**, and **marketers**.
  - Often make decisions based on these lies: "4 out of 5 dentists recommend...."



## To be clear...

- Many lies are unintentional
- People passing on misinformation/bad information that they don't even know is bad.
- People using bad data to make inferences
- People not understanding statistics well enough

• A way to make sense of information from data





- A way to make sense of information from data
- Framework for thinking, for reaching insights, for understanding the universe, for finding patterns and for solving problems.



- A way to make sense of information from data
- Framework for thinking, for reaching insights, for understanding the universe, for finding patterns and for solving problems.
- Numbers alone mean very little without context

=mX+0



- A way to make sense of information from data
- Framework for thinking, for reaching insights, and solving problems.
- Numbers alone mean very little without context
- Statistics is a marriage of:
  - Math
  - Science
  - Art



### Random Quote

"Statistical Thinking will one day be as necessary for efficient citizenship as the ability to read and write"

- H.G. Wells



## **Statistical Inference**



 Making an estimate or prediction about a <u>population</u> based on a <u>sample</u>.

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  - Often very expensive/impossible to survey an entire population (all students at UW, all residents in the U.S)

## **Statistical Inference**



- Making an estimate or prediction about a <u>population</u> based on a <u>sample</u>.
  - Often very expensive/impossible to survey an entire population (all students at UW, all residents in the U.S)
  - Need to use a random unbiased sample of population to draw conclusions (with some chance/margin of error)

"The Literary Digest" Magazine wanted to predict 1936 election:

- Alfred Landon vs Franklin D Roosevelt
- Sent 10 million surveys and received 2.4 million responses
- From a "List" containing: their subscribers, owners of cars and telephones

Electoral Votes	Prediction	Actual
Landon		
Roosevelt		



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What went wrong?





- Not Representative
  - Voluntary Response Bias
    - Only 24% of respondents answered the poll.



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  - Convenience Sampling
    - Only to people whose contact information they have.
    - Like standing outside a church and asking "Do you believe in God?", using those samples to represent the US population.



Let  $x_1, x_2, ..., x_n$  be <u>iid</u> samples...

- Not Representative
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  - Not the Right Population
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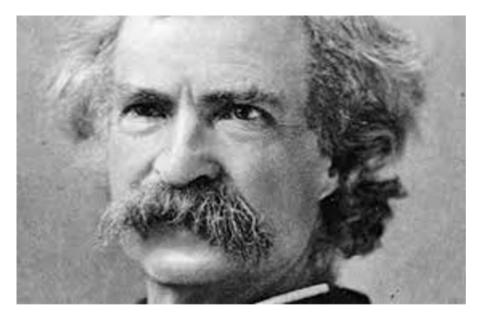
#### More samples is NOT a solution for bad sampling technique...



#### **Random Quote**

# "Facts are stubborn, but statistics are more pliable."

- Mark Twain



#### Random Quote

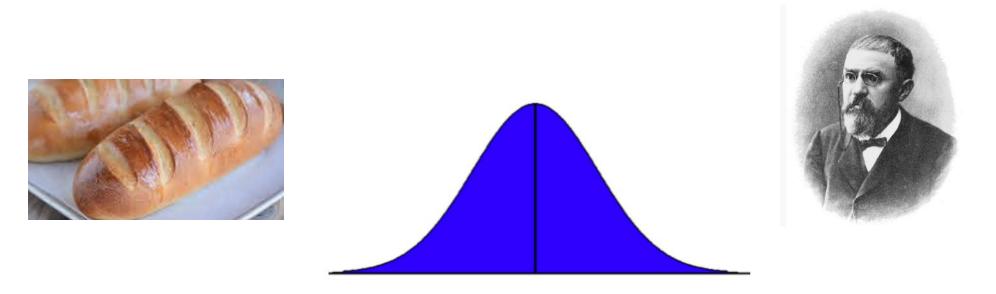
"82.123456789% of statistics are made up."

- Alex Tsun.



## 2. Detecting lies with statistics

A story about the famous French mathematician Henri Poincare

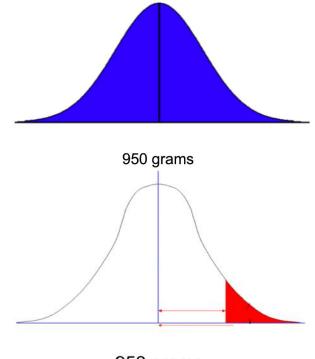


950 grams

# 2. Detecting lies with statistics

A story about the famous French mathematician Henri Poincare







950 grams

# To fake a distribution...

You'd better know what it looks like ....

People that are untrained in statistics often don't.

For example, people are really bad at faking a sequence of fair coin tosses.

"It's easy to lie with statistics. It's hard to tell the truth without statistics."

- Andrejs Dunkels



# First digit phenomenon

Suppose that I pick a random integer in the range 1..999

What's the chance that the first digit of the number I pick is a 1?

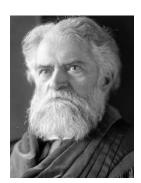
- a). About 1/9
- b). About 11%
- c) 30%
- d) I don't know.

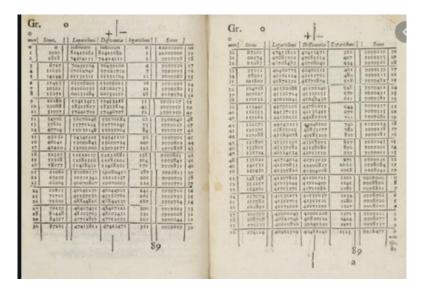
About 1/9, which is about 11%

# **Benford's Law**

How about in real life? Do certain digits in numbers collected randomly from the front pages of the newspaper or census statistics or from stock-market prices occur more often than others?

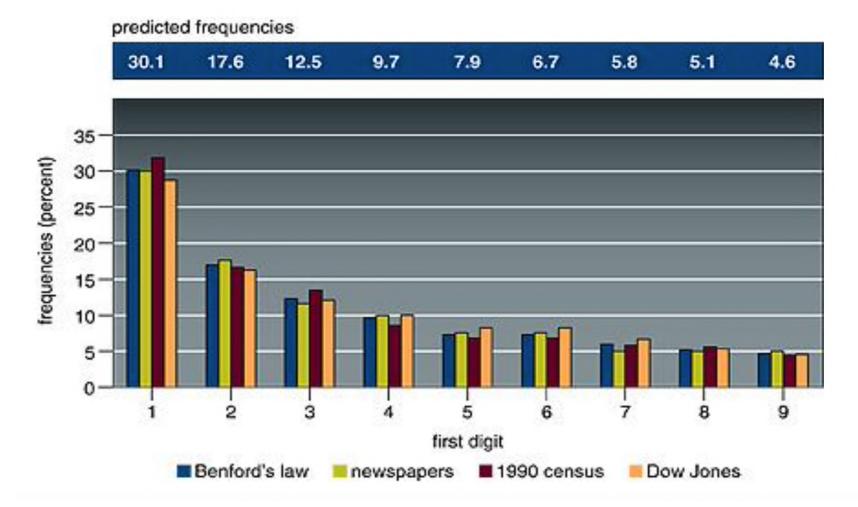
#### Frequency with which first significant digit is $d = \log (1 + 1/d)$







#### From "The First-Digit Phenomenon" by T. P. Hill, American Scientist, July-August 1998)



#### LONG-TERM EFFORTS TO "PROVE" BENFORD'S LAW

 What are the properties of a random sample of numbers that result in such a distribution? (e.g. not true for uniformly random numbers in 1..999)

#### • Scale invariance

- e.g. convert from dollars to pesos shouldn't change the first digit frequencies much)
- Independent of base

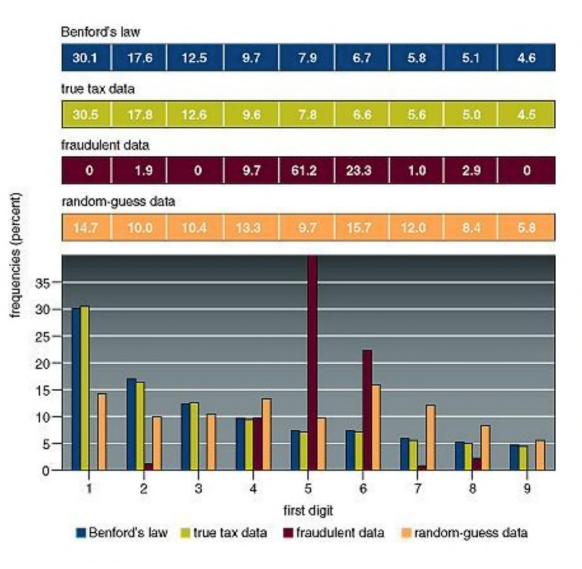
 $\circ~$  Equally valid when numbers expressed in base 10, base 100, or others

The only distributions on numbers that satisfies these conditions satisfy

Pr(first significant digit = d) = log (1 + 1/d)

## MODERN APPLICATION

 Using Benford's law to detect fraud or fabrication of data in financial documents.



"It is easy to lie with statistics, but easier to lie without them".

Fred Mosteller

#### "TOO GOOD TO BE TRUE"

- The special case of not appreciated the expected magnitude of sampling error.
- Data comes out "too good to be true", a telltale sign of having been tampered with, if not generated out of whole cloth.

#### GREGOR MENDEL'S SWEET PEAS

Postulated that self fertilization Of hybrid yellowseeded sweet peas would yield offspring with

Vellow

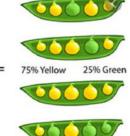
- 0.75 chance yellow-seeded
- 0.25 chance green seeded.

1865, reported results of 8023 experiments:

- 0.7505 yellow-seeded
- 0.2495 green-seeded.

Probability of observations this close to expected value is minute.





#### SOME TELLTALE SIGNS OF FAKERY....

- Wrong shape
- Too close to expected value (especially replicated)
- Too far from expected value
- Replications too good to be true.

Another famous example: Sir Cyril Burt's Twins

3 data sets: same to 3 decimal points.



# 3. p-Hacking

Manipulating data or statistical analyses to get **"significant p-values"** 

First, a brief primer on hypothesis testing and p-values.

Suppose that I believe that jelly beans cause acne. How might I provide evidence of this? Approach – "probabilistic proof by contradiction"



# **Hypothesis Testing**

Average teenager has amount of acne with mean  $\mu$  and variance  $\sigma^2$ 

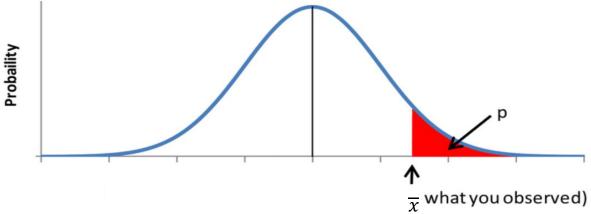
**H**<sub>0</sub> – null hypothesis (baseline): the mean amount of a has is μ, i. e. , jelly beans have no effect on acne

Want to provide evidence that the null hypothesis can be rejected!

**H**<sub>A</sub> - **Alternative hypothesis:** the mean amount of acne someone who eats jelly beans has is  $> \mu$ 

Choose **significance level**, say 0.05

Observe 100 jelly-bean-eating teenagers and measure their acne levels. Suppose sample mean observed  $\overline{x}$ 



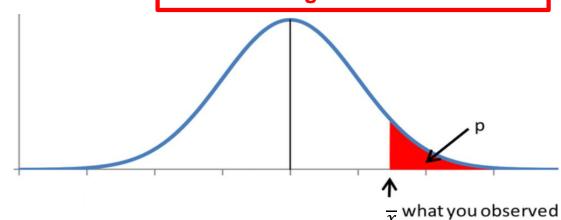
# **Hypothesis Testing**

Want to provide evidence that the null hypothesis can be rejected! at the 0.05 significance level

H<sub>0</sub> – null hypothesis (baseline): jelly beans have no effect on acne

H<sub>A</sub> - Alternative hypothesis: Jelly beans increase acne

Suppose find that for measured  $\overline{x}$ 



Pr (observing amount of acne this high if H<sub>0</sub> true) = Pr ( $\overline{X} \ge \overline{x}$ ) = 0.0162. This is our p-value.

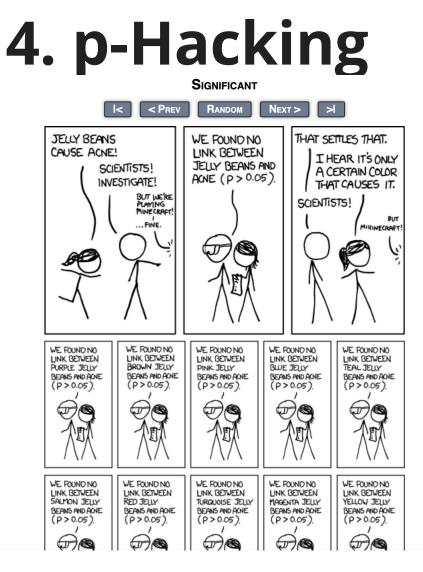
If p < 0.05 reject H<sub>0</sub> at the 0.05 significance level, i.e., strong statistical evidence that jelly beans cause an increase in acne. (If H<sub>0</sub> was true, this would be a very unlikely outcome).

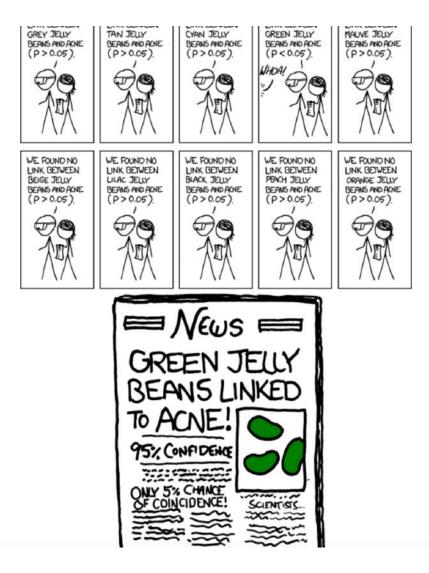
If p > 0.05, fail to reject  $H_{0}$ ; Not enough evidence to suggest the jelly bean effect on acne was significant.

Probaility

# p-Hacking

SIGNIFICANT RANDOM NEXT > > < Prev < JELLY BEANS WE FOUND NO THAT SETTLES THAT. CAUSE ACNE! LINK BETWEEN I HEAR IT'S ONLY JELLY BEANS AND SCIENTISTS! A CERTAIN COLOR ACNE (P>0.05). INVESTIGATE! THAT CAUSES IT. BUT WE'RE PLAYING MINE CRAFT! SCIENTISTS! 1 BUT ... FINE. MILLINECRAFT! WE FOUND NO LINK BETWEEN LINK BETWEEN LINK BETWEEN LINK BETWEEN LINK BETWEEN BROWN JELLY PURPLE JELLY PINK JELLY BLUE JELLY TEAL JELLY BEANS AND ACNE (P>0.05). (P>0.05) (P>0.05). (P>0.05). (P>0.05) 





# p-Hacking



- Scientists concluded that "Eating green jelly beans gives you more acne" after testing that teenagers who ate green jelly beans have more acne than those who don't, with a p-value of 0.05".
  - **The p-value means:** if the null hypothesis is true (teens who eat green jelly beans and those who don't have the **same** amount of acne), the probability of observing at least as extreme an outcome as we did is p.
  - **Putting it another way, a p-value of 0.05 means:** only a 5% chance of seeing this much acne if green jelly beans don't cause acne
  - But what if I repeat the experiment 20 times?
  - The chance that in 20 trials I will never get a p value < 0.05 is  $0.95^{20} \approx 0.358$

In other words 64% of the time one of these tests will be significant. This result has no significance! Happened by random chance!

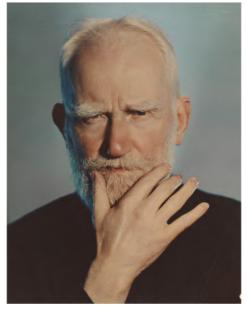
# 4. p-Hacking



- **Definition**: Performing the same hypothesis test multiple times in order to get a statistically significant result.
- The particularly evil thing: reporting only the significant tests, but not reporting the other 19 tests.....

"If at first you don't succeed, try two more times so your failure is statistically significant".

- George Bernard Shaw



"Torture numbers, and they'll confess to anything"

- George Easterbrook



#### Another interesting misuse of statistics

Attali/Bar-Hillel noticed that SAT answer keys are not randomized.

Keys are balanced rather than randomized.

Was easy for statisticians to detect by examining published tests.

This is a case of thinking "randomization is too important to be left to chance "!

#### Suggests a strategy for test-takers

- Answer all the questions you can.
- When guessing the rest, pick an answer position that occurs least frequently in your answers.

Simulations shows this adds 10-16 points over random guessing.

Claimed to be more gain than some very expensive SAT prep courses!

# Conclusions

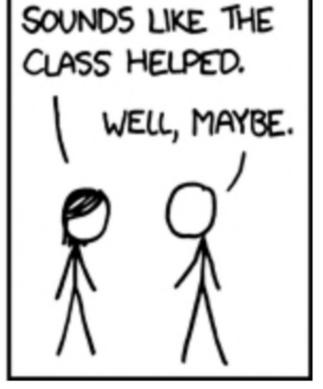
- 1. Determine if the samples are **random** and **representative**.
- 2. Ask for a confidence interval.
- 3. Be dubious. Be extremely dubious.
- 4. Don't make up data or statistics. You'll get caught.
- 5. Be wary of p-hacking (and don't do it yourself)!
- 6. Be careful about seeing patterns where there are none.
- 7. Correlation does not imply causation.

I USED TO THINK CORRELATION IMPLIED CAUSATION. 1



Source: https://xkcd.com/552/

THEN I TOOK A STATISTICS CLASS. NOW I DON'T.



# Conclusions

- 1. Determine if the samples are **random** and **representative**.
- 2. Ask for a confidence interval.
- 3. Be dubious. Be extremely dubious.
- 4. Don't make up statistics. You'll get caught.
- 5. Be wary of p-hacking (and don't do it yourself)!
- 6. Be careful about seeing patterns where there are none.
- 7. Correlation does not imply causation.
- 8. Interpret conditional probabilities properly. Intuition sometimes doesn't work here!
- 9. Be wary of assuming things are independent that aren't independent.

"Data is the sword of the 21st century, those who wield it well, the Samurai."

- Jonathan Rosenberg (ex-Google SVP)





#### "Do not trust any statistics you did not fake yourself"

- Winston Churchill

