A glimpse of auction theory

Anna Karlin

Agenda

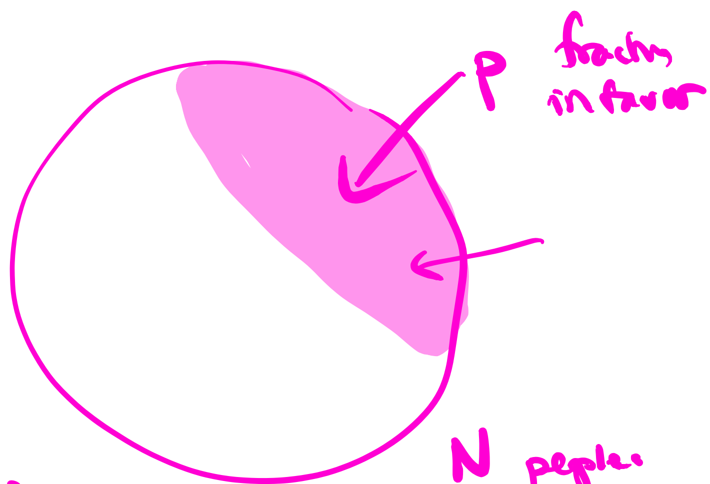
- Loose end – continuity correction
- A glimpse of auction theory

Polling

want to estimate p.

$$X_i = \begin{cases} 1 & \text{in favor} \\ 0 & \text{not.} \end{cases}$$

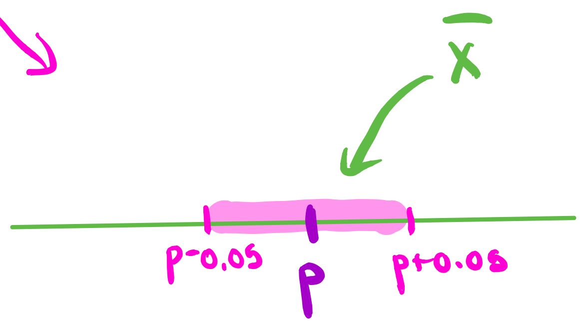
$X_i \sim \mathrm{Ber}(p)$



$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ sample mean. $\Rightarrow$ estimate.

how big does n need to be to guarantee "good" estimate?

CLT $\quad \bar{X} \sim N\left(p, \frac{p(1-p)}{n}\right)$



If $n \geq 543$, then we can be 98% confident that our poll result is within 5% of true p

Called a "confidence interval"

statistics

once I look at value of X:
called a "sample"

$\overline{X}$ sample mean

=

The Continuity Correction (Idea)

Suppose want to use CLT to estimate $\Pr(28 \le X \le 30)$ when X is Binomial (100, 0.3)

$$\boxed{X = \sum_{i=1}^{100} X_i} \qquad X_i \sim Ber(0.3)$$

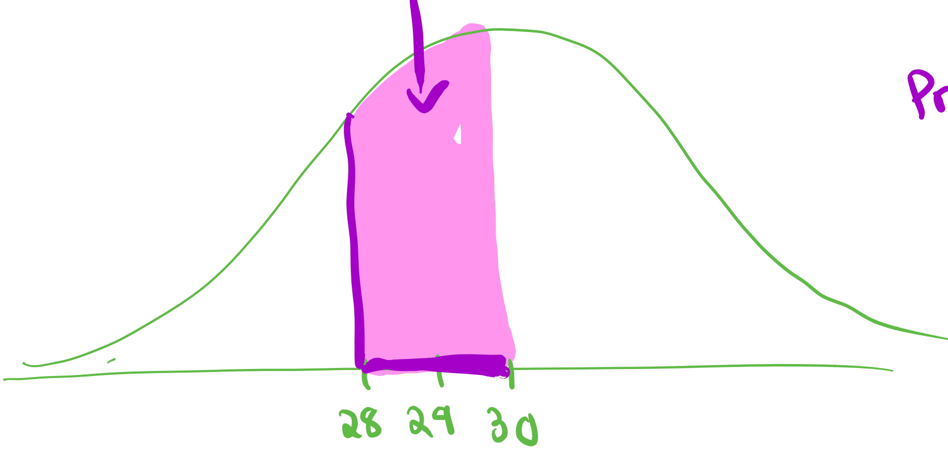
Issue: Binomial is discrete, Normal is continuous.

$$X \text{ approx } \boxed{N(30, 100 \cdot 0.3 \cdot 0.7)}$$

$$\text{Estimate } \Pr(28 \le X \le 30) = [P_X(28) + P_X(29) + P_X(30)]$$

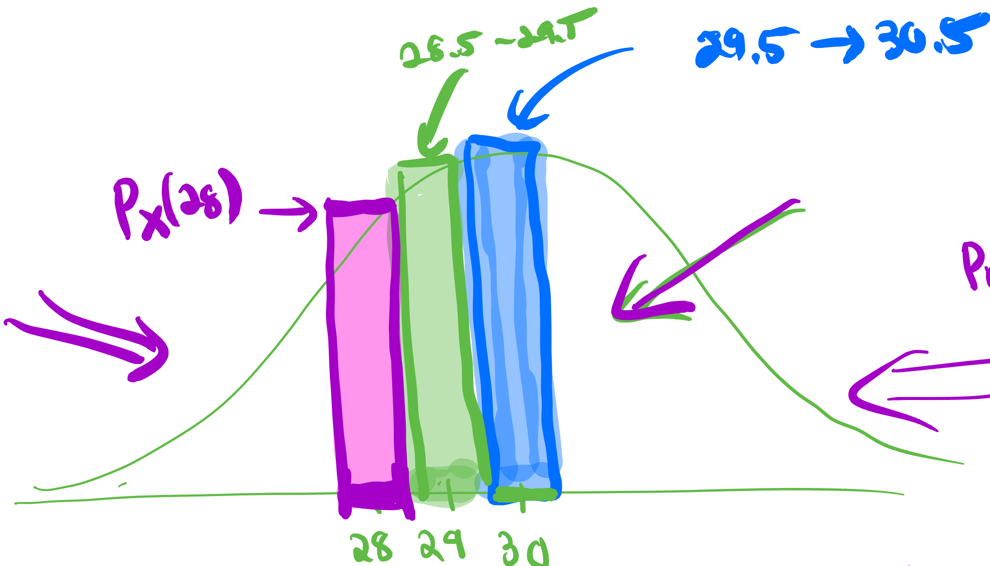
pmf of $Bin(100, 0.3)$

$$W \sim N(30, 100 \cdot 0.3 \cdot 0.7)$$



$\cancel{Pr(28 \le W \le 30)}$

$Pr(27.5 \le W \le 30.5)$



continuity correction.

$$Pr(a \le X \le b)$$

$$\approx Pr(a-0.5 \le W \le b+0.5)$$

AUCTIONS

- Companies like Google and Facebook make most of their money by selling ads.
- The ads are sold via auction.

Facebook Ads bidding... 🤔 Is this an auction?

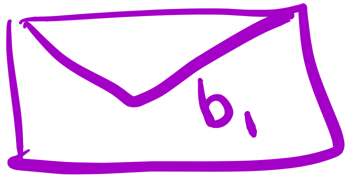
Yes! That's the first thing you need to understand to master bidding management of Facebook Ads. **When you're creating a new campaign, you're joining a huge, worldwide auction.**

You'll be competing with hundreds of thousands of advertisers to buy what Facebook is selling: Real estate on the News Feed, Messenger, Audience Network, and mobile apps to display your ads to the users.

An auction is a ...

- Game
  - Players: advertisers
  - Strategy choices for each player: possible bids
  - Rules of the game – made up by Google/Facebook/whoever is running the auction

- What do we expect to happen? How do we analyze mathematically?



Special case: Sealed Bid single item auction

- Say I decide to run an auction to sell my laptop and I let you be the bidders.
- If I want to make as much money as possible - what should the rules of the auction be?

Some possibilities:

- **First price auction:** highest bidder wins; pays what they bid.
- **Second price auction:** highest bidder wins; pays second highest bid.
- **All pay auction:** highest bidder wins: all bidders pay what they bid.

SEALED BID SINGLE ITEM AUCTION

Some possibilities:

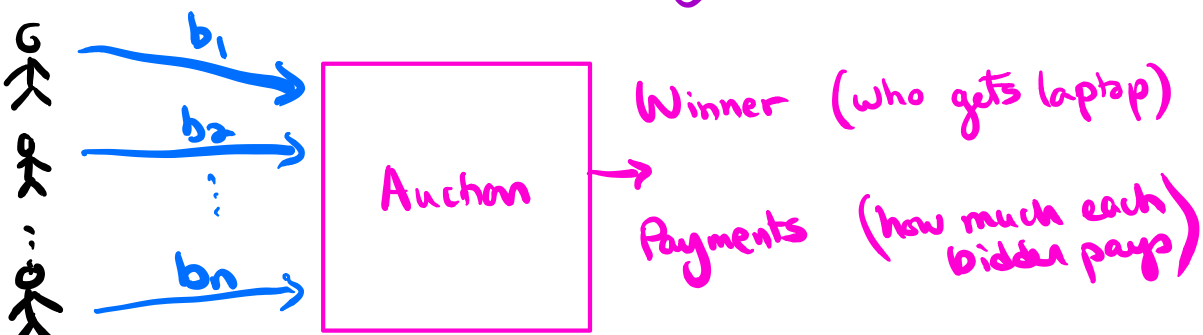
a) - **First price auction:** highest bidder wins; pays what they bid.
b) - **Second price auction:** highest bidder wins; pays second highest bid.
c) - **All pay auction:** highest bidder wins: all bidders pay what they bid.

Which of these will make me the most money?

100 61 48 2

FP	100 wins	pays 100
2ndP	100 wins	61
⇒ Allpay	100 wins	everybody pays bid

⇓



$ 100

Bidder model

Each bidder has a value, say v_i for bidder i.

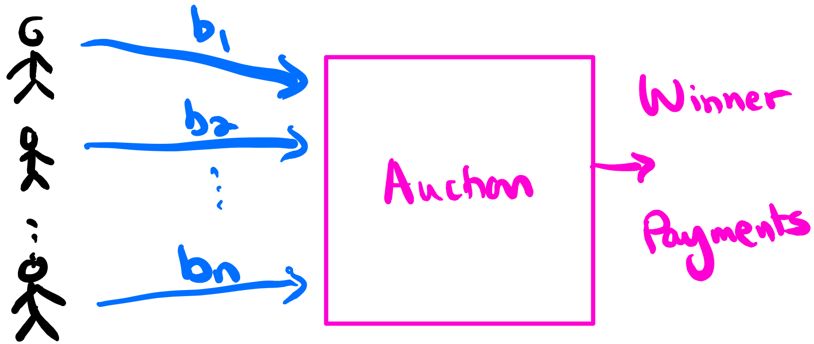
Bidder is trying to maximize their "utility" –
the value of the item they get – price they pay.

V $100
pay $80
⇒ utility $20

V $100
pay $101
⇒ utility $-1

v $100
bid $90

uhhh 0.



100 61 48 2

bid the value

other bids 61, 48, 2

a) $100
b) $90
c) $61.01
d) $59

⇒ FP 100 wins pays 100 (c)

2nd P 100 wins 61 (a)

→ Allpay 100 wins everybody pays bid

Thm 2nd price auction is truthful. [It's always in my best interest to bid my value]

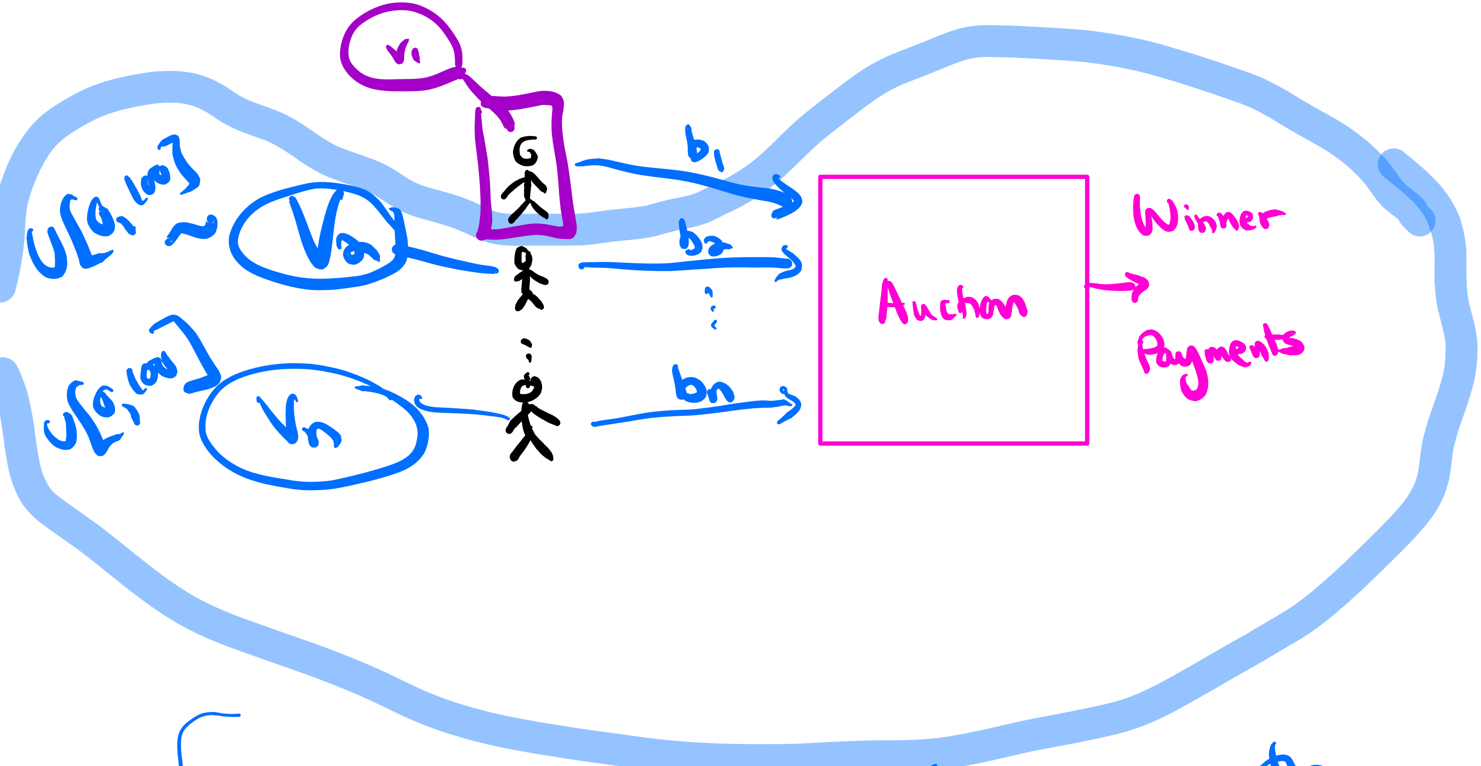
FP, Allpay not truthful auctions.

$V_1 \sim U[0,100]$ $\quad$ v_1 sample from distn.

$V_2 \sim U[0,100]$

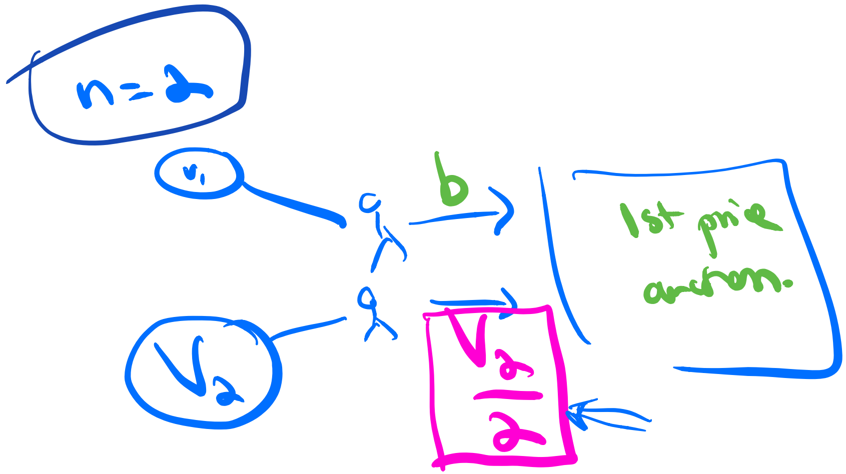
$\vdots$

$V_n \sim U[0,100]$



best response in expectation over the randomness in other people's values $v_2 \dots v_n$

Bayes-Nash eq.



$$E(\text{utility of bid } b) = (v_1 - b)\Pr(\text{win})$$

$$= (v_1 - b)\Pr\left(b > \frac{V_2}{2}\right)$$

$$= (v_1 - b)\underbrace{\Pr(V_2 < 2b)}_{= F_{V_2}(2b) = \frac{2b}{100} = \frac{b}{50}}$$

$$V_2 \sim U[0, 100]$$

$$F_{V_2}(x) = \frac{x}{100}$$

$$= \underbrace{(v_1 - b)\frac{b}{50}}$$

choose b to maximize this

deriv wrt b.

$$\underbrace{-\frac{b}{50} + \frac{v_1 - b}{50}} = 0$$

$$v_1 - 2b = 0$$

$$\boxed{b = \frac{v_1}{2}}$$

bidding half your value is a Bayes-Nash equilibrium

In this situation

exp auctioneer revenue $= E\left[\max\left(\frac{v_1}{2}, \frac{v_2}{2}\right)\right]$

$= \frac{1}{2} E\left[\max(v_1, v_2)\right]$