Polling + Auctions

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Agenda

- An application of The Central Limit Theorem – Polling
- A glimpse of auction theory

Magic mushrooms

Yesterday, Oregonians are voting on whether to legalize the therapeutic use of “magic mushrooms”.

If you take a "heroic" dose, supposedly, “the ego dissolves and the user feels inseparable from the rest of the universe.”

CLT → Polling on magic mushrooms

The Central Limit Theorem

Consider i.i.d. (independent, identically distributed) random vars
$X_1, X_2, X_3, \ldots$
Where X_i has $\mu = E[X_i]$ and $\sigma^2 = Var[X_i]$

As $n \to \infty$,

$$\frac{X_1+X_2+\cdots+X_n-n\mu}{\sigma\sqrt{n}} \longrightarrow N(0,1)$$

Restated: As $n \to \infty$,

$$\frac{1}{n}\sum_{i=1}^{n} X_i \to N\left(\mu, \frac{\sigma^2}{n}\right)$$

The Standard Normal CDF

Φ Table: $\mathbb{P}(Z \leq z)$ when $Z \sim \mathcal{N}(0,1)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.5279	0.53188	0.53586
0.1	0.53983	0.5438	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.6293	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.6591	0.66276	0.6664	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.7054	0.70884	0.71226	0.71566	0.71904	0.7224
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.7549
0.7	0.75804	0.76115	0.76424	0.7673	0.77035	0.77337	0.77637	0.77935	0.7823	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.8665	0.86864	0.87076	0.87286	0.87493	0.87698	0.879	0.881	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.9032	0.9049	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.9222	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.9452	0.9463	0.94738	0.94845	0.9495	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.9608	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.9732	0.97381	0.97441	0.975	0.97558	0.97615	0.9767
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.9803	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.983	0.98341	0.98382	0.98422	0.98461	0.985	0.98537	0.98574
2.2	0.9861	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.9884	0.9887	0.98899
2.3	0.98928	0.98956	0.98983	0.9901	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.9918	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.9943	0.99446	0.99461	0.99477	0.99492	0.99506	0.9952
2.6	0.99534	0.99547	0.9956	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.9972	0.99728	0.99736
2.8	0.99744	0.99752	0.9976	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.999

The Continuity Correction (Idea)

Suppose want to use CLT to estimate $\Pr(28 \leq X \leq 30)$ when X is Binomial (100, 0.3)

Issue: Binomial is discrete, Normal is continuous.

Auctions

- Companies like Google and Facebook make most of their money by selling ads.
- The ads are sold via auction.

Facebook Ads bidding... 🤔 Is this an auction?

Yes! That's the first thing you need to understand to master bidding management of Facebook Ads. **When you're creating a new campaign, you're joining a huge, worldwide auction.**

You'll be competing with hundreds of thousands of advertisers to buy what Facebook is selling: Real estate on the News Feed, Messenger, Audience Network, and mobile apps to display your ads to the users.

An auction is a ...

- Game
  - Players: advertisers
  - Strategy choices for each player: possible bids
  - Rules of the game – made up by Google/Facebook/whoever is running the auction

- What do we expect to happen? How do we analyze mathematically?

Special case: Sealed Bid single item auction

- Say I decide to run an auction to sell my laptop and I let you be the bidders.
- If I want to make as much money as possible - what should the rules of the auction be?

Some possibilities:

- **First price auction:** highest bidder wins; pays what they bid.
- **Second price auction:** highest bidder wins; pays second highest bid.
- **All pay auction:** highest bidder wins: all bidders pay what they bid.

SEALED BID SINGLE ITEM AUCTION

Some possibilities:

- **First price auction:** highest bidder wins; pays what they bid.
- **Second price auction:** highest bidder wins; pays second highest bid.
- **All pay auction:** highest bidder wins: all bidders pay what they bid.

Which of these will make me the most money?

Bidder model

Each bidder has a value, say v_i for bidder i.

Bidder is trying to maximize their "utility" –
the value of the item they get – price they pay.

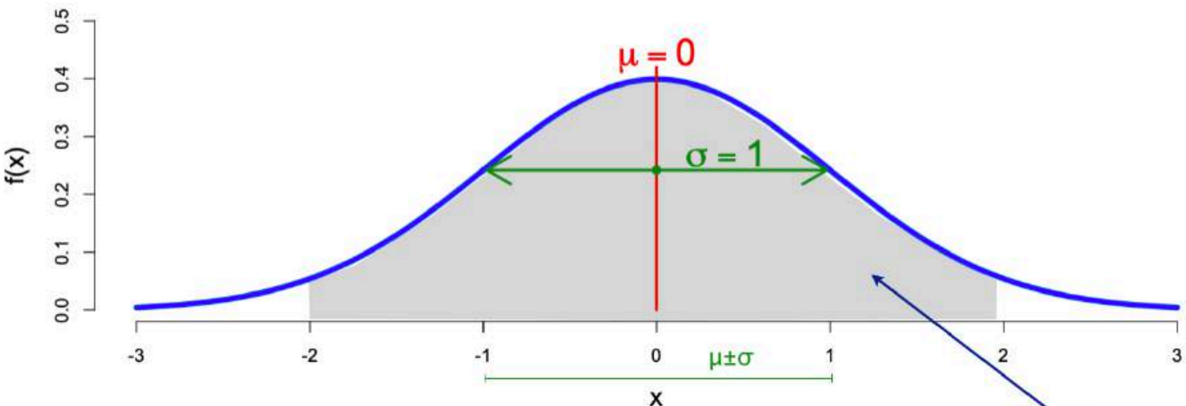
The Central Limit Theorem

Central Limit Theorem (CLT): Let $X_1, \ldots, X_n$ be a sequence of iid random variables with mean μ and (finite) variance σ^2. We've seen that the sample mean $\bar{X}_n$ has mean μ and variance σ^2/n. Then, as $n \to \infty$, the following equivalent statements hold:

1. $\bar{X}_n \to \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$.
2. $\frac{\bar{X}_n - \mu}{\sqrt{\sigma^2/n}} \to \mathcal{N}(0, 1)$.
3. $\sum_{i=1}^{n} X_i \sim \mathcal{N}(n\mu, n\sigma^2)$. (Not "technically" correct, but useful for applications).
4. $\frac{\sum_{i=1}^{n} X_i - n\mu}{\sqrt{n\sigma^2}} \to \mathcal{N}(0, 1)$.

The mean or variance are no surprise; the importance of the CLT is, regardless of the distribution of the X_i's, the sample mean approaches a Normal distribution as $n \to \infty$.

Normal random variables



If $Z \sim N(\mu,\sigma^2)$ what is $P(\ \mu-\sigma < Z < \mu+\sigma\ )$?

$P(\ \mu - \ \sigma < Z < \mu + \ \sigma\ ) = \Phi(1) - \Phi(-1) \approx 68\%$

$P(\ \mu - 2\sigma < Z < \mu + 2\sigma\ ) = \Phi(2) - \Phi(-2) \approx 95\%$

$P(\ \mu - 3\sigma < Z < \mu + 3\sigma\ ) = \Phi(3) - \Phi(-3) \approx 99\%$

Why?

$N(\mu,\sigma^2)$ $N(0,1)$

$\mu - k\sigma < \boxed{Z} < \mu + k\sigma \quad \Leftrightarrow \quad -k < \boxed{(Z-\mu)/\sigma} < +k$

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2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
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