Announcements

- Homework 1 due yesterday
- Homework 2 due next Wednesday (10/14) 11:59 pm PST
Review

- Some important denotation and definition on your handout
- Countable additivity: only apply when events are **mutually exclusive / disjoint**
- Inclusion-Exclusion: +singles - doubles + triples - quads + ...

<table>
<thead>
<tr>
<th>Theorem 2.1.4: Probability in Sample Space with Equally Likely Outcomes</th>
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| If $\Omega$ is a sample space such that each of the unique outcome elements in $\Omega$ are **equally likely**, then for any event $E \subseteq \Omega$:
| $P(E) = \frac{|E|}{|\Omega|}$ |
The MatPlotLib Library
Plotting A Graph using matplotlib.pyplot

```python
x = np.arange(10)
y = x ** 2
z = 5*x + 7
plt.plot(x, y, "b", label="y = x^2", linestyle='--')
```

- The `x` and `y` coordinates of the data
- Line color. Some abbreviations available, such as `r` - red, `g` - green, `b` - blue, etc.
- Label for line in the legend
- Line style. `-' gives a solid line, `--` gives a dashed one, `'-.'` gives a dash-dot one, etc.
import matplotlib.pyplot as plt
import numpy as np

x = np.arange(10)
y = x ** 2
z = 5*x + 7
plt.plot(x, y, "b", label="y = x^2", linestyle='--')
plt.plot(x, z, "r", label="z = 5x + 7", linestyle='-.')
plt.legend(loc="upper left")
plt.xlabel("x")
plt.ylabel("f(x)")
plt.title("An Interesting Graph")
plt.savefig('plot.png')
$P(E)$

The long-term limit of probability of an event $E$ occurring in a random experiment

$$\frac{\text{# of trials (}E\text{)}}{\text{# trials}} \to P(E)$$
A Coin Flip Game

Suppose a weighted coin comes up heads with probability \( \frac{1}{3} \).

How many flips do you think it will take for the first head to appear?
Simulating the Coin Flip Game

np.random.rand()  →  Returns a single random float in the range [0, 1)
Simulating the Coin Flip Game

What is this expression checking?

```python
if np.random.rand() < p:
```

Since `np.random.rand()` returns a random float between `[0, 1)`, the function returns a value `< p` with probability `p`. 
Simulating the Coin Flip Game

What is this expression checking?

Since `np.random.rand()` returns a random float between \([0, 1)\), the function returns a value \(< p\) with probability \(p\).

This allows us to simulate the event in question: the first ‘Heads’ appears whenever `rand()` returns a value \(< p\).

And, if `rand()` \(\geq p\), the coin flip turned up ‘Tails’.
Simulating ONE Coin Flip Game

```python
def sim_one_game():
    flips = 0
    while True:
        flips += 1
        if np.random.rand() < p:
            return flips
```

Counter that keeps track of number of coin flips

When we “flip a head”, we return the total number of times we’ve flipped the coin.
import numpy as np

def coin_flips(p: float = 1/3, ntrials: int = 5000) -> float:
    def sim_one_game() -> int:
        flips = 0
        while True:
            flips += 1
            if np.random.random() < p:
                return flips
        total_flips = 0
        for i in range(ntrials):
            total_flips += sim_one_game()
        return total_flips / ntrials
Helper function simulates one game

```
import numpy as np

def coin_flips(p: float = 1/3, ntrials: int = 5000) -> float:
    def sim_one_game() -> int:
        flips = 0
        while True:
            flips += 1
            if np.random.rand() < p:
                return flips

    total_flips = 0
    for i in range(ntrials):
        total_flips += sim_one_game()
    return total_flips / ntrials
```

After each game, adds the total number of flips taken.

Finally, we return the average # of flips it took for the first H to appear.
Codealong: Probability by Simulation
Question 3: “Spades and Hearts”

Given 3 different spades and 3 different hearts, shuffle them. Compute $Pr(E)$, where $E$ is the event that the suits of the shuffled cards are in alternating order.
If $\Omega$ is a sample space such that each of the unique outcome elements in $\Omega$ are equally likely, then for any event $E \subseteq \Omega$:

$$P(E) = \frac{|E|}{|\Omega|}$$

Computing probability in the case of **equally likely outcomes** reduces to doing two counting problems (**counting $|E|$ and $|\Omega|$**, where computing $|\Omega|$ is generally easier than computing $|E|$). Just use the techniques from Chapter 1 (Counting) to do this!

-Textbook
Size of sample space: all reorderings possible

6!
Size of sample space: all reorderings possible

6!

Size of event:

3! ways to order spades, 3! ways to order hearts either hearts at the front or spades at the front

\[2 \times 3!^2\]
Size of sample space: all reorderings possible

\( 6! \)

Size of event:

3! ways to order spades, 3! ways to order hearts either hearts at the front or spades at the front

Answer: \( \frac{2 \times 3!^2}{6} \)
Question 4: “Trick or Treat”

Suppose on Halloween, someone is too lazy to keep answering the door, and leaves a jar of exactly $N$ total candies. You count that there are exactly $K$ of them which are kit kats (and the rest are not). The sign says to please take exactly $n$ candies. Each item is equally likely to be drawn. Let $X$ be the number of kit kats we draw (out of $n$). What is $Pr(X = k)$, that is, the probability we draw exactly $k$ kit kats?
If $\Omega$ is a sample space such that each of the unique outcome elements in $\Omega$ are equally likely, then for any event $E \subseteq \Omega$:

$$P(E) = \frac{|E|}{|\Omega|}$$

Computing probability in the case of **equally likely outcomes** reduces to doing two counting problems (counting $|E|$ and $|\Omega|$), where computing $|\Omega|$ is generally easier than computing $|E|$). Just use the techniques from Chapter 1 (Counting) to do this!

-Textbook
Size of Sample Space: the total number of ways to choose $n$ candies out of $N$ total.
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$$\Pr(X = k) = \frac{|E|}{\binom{N}{n}}$$
Size of Sample Space: the total number of ways to choose \(n\) candies out of \(N\) total.

Size of Event: counted in two stages!

\[
\Pr(X = k) = \frac{|E|}{\binom{N}{n}}
\]
Size of Sample Space: the total number of ways to choose \( n \) candies out of \( N \) total.

Size of Event: counted in two stages!

\[
Pr(X = k) = \binom{K}{k} \binom{N - K}{n - k} / \binom{N}{n}
\]

1. choose \( k \) out of the \( K \) kit kats
2. Then choose \( n - k \) out of the \( N - K \) other candies
Question 6: “Weighed Die”

Consider a weighted (6-faced) die such that

- $\Pr(1) = \Pr(2)$,
- $\Pr(3) = \Pr(4) = \Pr(5) = \Pr(6)$, and
- $\Pr(1) = 3\Pr(3)$.

What is the probability that the outcome is [3 or 4]?
• Pr(1) = Pr(2)
• Pr(3) = Pr(4) = Pr(5) = Pr(6)
• Pr(1) = 3Pr(3)

the sum of probabilities for the sample space must equal 1
• $Pr(1) = Pr(2)$
• $Pr(3) = Pr(4) = Pr(5) = Pr(6)$
• $Pr(1) = 3Pr(3)$

the sum of probabilities for the sample space must equal 1

$Pr(1) + Pr(2) + Pr(3) + Pr(4) + Pr(5) + Pr(6) = 1$
- $\Pr(1) = \Pr(2)$
- $\Pr(3) = \Pr(4) = \Pr(5) = \Pr(6)$
- $\Pr(1) = 3\Pr(3)$

the sum of probabilities for the sample space must equal 1

$\Pr(1) + \Pr(2) + \Pr(3) + \Pr(4) + \Pr(5) + \Pr(6) = 1$

Use the given equations to substitute everything into $\Pr(3)$:

$3\Pr(3) + 3\Pr(3) + \Pr(3) + \Pr(3) + \Pr(3) + \Pr(3) = 10\Pr(3) = 1$
• \( \Pr(3) = 0.1 \)

\( \Pr(3) = \Pr(4) = 0.1 \)
- $Pr(3) = 0.1$

$Pr(3) = Pr(4) = 0.1$

$Pr(3 \text{ or } 4) = Pr(3) + Pr(4) = 0.2$