Section 2 Slides

Made by Leiyi Zhang, Scott Ni and Shreya Jayaraman

Announcements

- Homework 1 due yesterday
- Homework 2 due next Wednesday (10/14) 11:59 pm PST
-

Review

- Some important denotation and definition on your handout
- Countable additivity: only apply when events are mutually exclusive / disjoint
- Inclusion-Exclusion: +singles - doubles + triples - quads + …
- **Theorem 2.1.4: Probability in Sample Space with Equally Likely Outcomes**

  If Ω is a sample space such that each of the unique outcome elements in Ω **are equally likely**, then for any event $E \subseteq \Omega$:

  $$\mathbb{P}(E) = \frac{|E|}{|\Omega|}$$

The MatPlotLib Library

Plotting A Graph using matplotlib.pyplot

```
x = np.arange(10)
y = x ** 2
z = 5*x + 7
plt.plot(x, y, "b", label="y = x^2", linestyle='-')
```

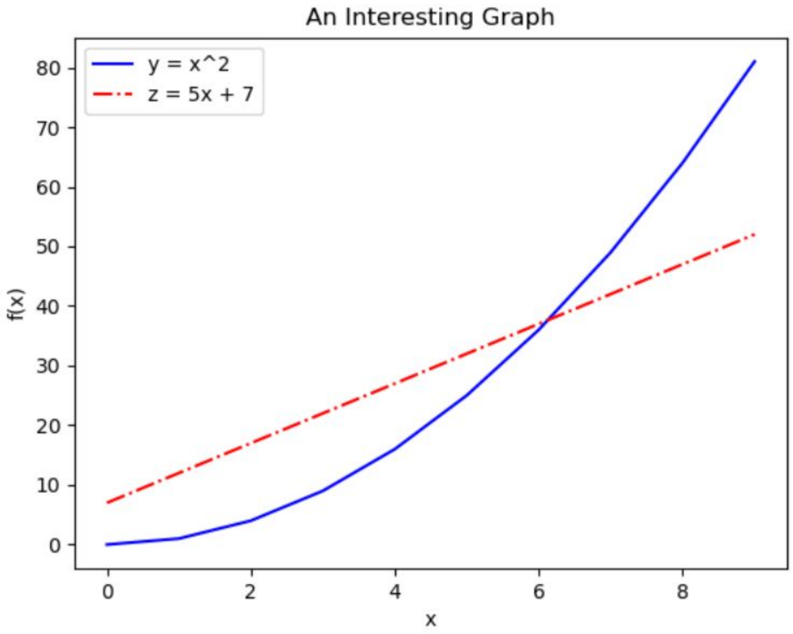
The x and y coordinates of the data

Line color. Some abbreviations available, such as r - red, g - green, b - blue, etc.

Label for line in the legend

Line style. ‘-’ gives a solid line, ‘--’ gives a dashed one, ‘-.’ gives a dash-dot one, etc.

```python
import matplotlib.pyplot as plt
import numpy as np

x = np.arange(10)
y = x ** 2
z = 5*x + 7
plt.plot(x, y, "b", label="y = x^2", linestyle='-')
plt.plot(x, z, "r", label="z = 5x + 7", linestyle='-.')
plt.legend(loc="upper left")
plt.xlabel("x")
plt.ylabel("f(x)")
plt.title("An Interesting Graph")
plt.savefig('plot.png')
```



Probability by Simulation

P(E)

The long-term limit of probability of an event E occurring in a random experiment

$$\frac{\#\ of\ trials\ (E)}{\#trials} \rightarrow P(E)$$

A Coin Flip Game

Suppose a weighted coin comes up heads with probability ⅓.

How many flips do you think it will take for the first head to appear?

Simulating the Coin Flip Game

`np.random.rand()` → Returns a single random float in the range [0, 1)

Simulating the Coin Flip Game

```
if np.random.rand() < p:
```

What is this expression checking?

Since np.random.rand() returns a random float between [0, 1), the function returns a value < p with probability p.

Simulating the Coin Flip Game

```
if np.random.rand() < p:
```

What is this expression checking?

Since np.random.rand() returns a random float between [0, 1), the function returns a value < p with probability p.

This allows us to simulate the event in question: **the first 'Heads' appears whenever rand() returns a value < p.**
And, if rand() >= p, the coin flip turned up 'Tails'.

Simulating ONE Coin Flip Game

```python
def sim_one_game():
    flips = 0
    while True:
        flips += 1
        if np.random.rand() < p:
            return flips
```

Counter that keeps track of number of coin flips

When we “flip a head”, we return the total number of times we’ve flipped the coin.

P(heads)

Number of games we want to simulate

Helper function simulates one game

```
import numpy as np

def coin_flips(p:float = 1/3, ntrials:int = 5000) -> float:
    def sim_one_game() -> int:
        flips = 0
        while True:
            flips += 1
            if np.random.rand() < p:
                return flips

    total_flips = 0
    for i in range(ntrials):
        total_flips += sim_one_game()
    return total_flips / ntrials
```

P(heads)

Number of games we want to simulate

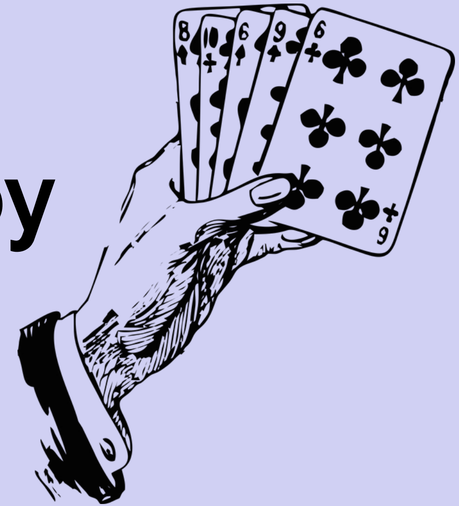
```python
import numpy as np

def coin_flips(p:float = 1/3, ntrials:int = 5000) -> float:
    def sim_one_game() -> int:
        flips = 0
        while True:
            flips += 1
            if np.random.rand() < p:
                return flips

    total_flips = 0
    for i in range(ntrials):
        total_flips += sim_one_game()
    return total_flips / ntrials
```

Helper function simulates one game

After each game, adds the total number of flips taken.

Finally, we return the average # of flips it took for the first H to appear

Codealong: Probability by Simulation



Question 3: “Spades and Hearts”

Given 3 different spades and 3 different hearts, shuffle them. Compute Pr(E), where E is the event that the suits of the shuffled cards are in alternating order.

If Ω is a sample space such that each of the unique outcome elements in Ω **are equally likely**, then for any event $E \subseteq \Omega$:

$$\mathbb{P}(E) = \frac{|E|}{|\Omega|}$$

Computing probability in the case of **equally likely outcomes** reduces to doing two counting problems (counting $|E|$ and $|\Omega|$, where computing $|\Omega|$ is generally easier than computing $|E|$). Just use the techniques from Chapter 1 (Counting) to do this!

-Textbook

Size of sample space: all reorderings possible

6!

Size of sample space: all reorderings possible

6!

Size of event:

3! ways to order spades, 3! ways to order hearts
either hearts at the front or spades at the front

$$2 * 3!^2$$

Size of sample space: all reorderings possible

6!

Size of event:

3! ways to order spades, 3! ways to order hearts
either hearts at the front or spades at the front

$$2 * 3!^2$$

Answer :

$$\frac{2 * 3!^2}{6}$$

Question 4: “Trick or Treat”

Suppose on Halloween, someone is too lazy to keep answering the door, and leaves a jar of exactly N total candies. You count that there are exactly K of them which are kit kats (and the rest are not). The sign says to please take exactly n candies. Each item is equally likely to be drawn. Let X be the number of kit kats we draw (out of n). What is $Pr(X = k)$, that is, the probability we draw exactly k kit kats?

If Ω is a sample space such that each of the unique outcome elements in Ω **are equally likely**, then for any event $E \subseteq \Omega$:

$$\mathbb{P}(E) = \frac{|E|}{|\Omega|}$$

Computing probability in the case of **equally likely outcomes** reduces to doing two counting problems (counting $|E|$ and $|\Omega|$, where computing $|\Omega|$ is generally easier than computing $|E|$). Just use the techniques from Chapter 1 (Counting) to do this!

-Textbook

$$\Pr(X = k) = \frac{|E|}{|\Omega|}$$

Size of Sample Space: the total number of ways to choose **n** candies out of **N** total.

$$\Pr(X = k) = \frac{|E|}{\binom{N}{n}}$$

Size of Sample Space: the total number of ways to choose **n** candies out of **N** total.

Size of Event: counted in two stages!

$$\Pr(X = k) = \frac{|E|}{\binom{N}{n}}$$

Size of Sample Space: the total number of ways to choose **n** candies out of **N** total.

1. choose **k** out of the **K** kit kats

2. Then choose **n - k** out of the **N - K** other candies

Size of Event: counted in two stages!

$$\Pr(X = k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$$

Size of Sample Space: the total number of ways to choose **n** candies out of **N** total.

Question 6: “Weighed Die”

Consider a weighted (6-faced) die such that

- Pr(1) = Pr(2),
- Pr(3) = Pr(4) = Pr(5) = Pr(6), and
- Pr(1) = 3Pr(3).

What is the probability that the outcome is [3 or 4]?

- Pr(1) = Pr(2)
- Pr(3) = Pr(4) = Pr(5) = Pr(6)
- Pr(1) = 3Pr(3)

the sum of probabilities for the sample space must equal 1

- $Pr(1) = Pr(2)$
- $Pr(3) = Pr(4) = Pr(5) = Pr(6)$
- $Pr(1) = 3Pr(3)$

the sum of probabilities for the sample space must equal 1

$$Pr(1) + Pr(2) + Pr(3) + Pr(4) + Pr(5) + Pr(6) = 1$$

- $Pr(1) = Pr(2)$
- $Pr(3) = Pr(4) = Pr(5) = Pr(6)$
- $Pr(1) = 3Pr(3)$

the sum of probabilities for the sample space must equal 1

$Pr(1) + Pr(2) + Pr(3) + Pr(4) + Pr(5) + Pr(6) = 1$

Use the given equations to substitute everything into Pr(3):

$3Pr(3) + 3Pr(3) + Pr(3) + Pr(3) + Pr(3) + Pr(3) = 10Pr(3) = 1$

- $Pr(3) = 0.1$

$Pr(3) = Pr(4) = 0.1$

- Pr(3) = 0.1

Pr(3) = Pr(4) = 0.1

Pr(3 or 4) = Pr(3) + Pr(4) = 0.2