

Problem Set 1 (due Wednesday, October 7 at 11:59pm)

Directions: For each problem, remember you must briefly explain/justify how you obtained your answer, as correct answers without an explanation will receive **no credit**. Moreover, in the event of an incorrect answer, we can still try to give you partial credit based on the explanation you provide. Unless you are asked to, you should leave your answer in terms of factorials, combinations, etc., for instance 26^7 or $26!/7!$ or $26 \cdot \binom{26}{7}$.

Instructions as to how to upload your solutions to Gradescope are on the course web page.

Submission: You must upload a **pdf** of your solutions to Gradescope under "PSet 1". (Instructions as to how to upload your solutions to Gradescope are on the course web page.) The use of latex is highly recommended. (Note that if you want to hand-write your solutions, you'll need to scan them. We will take off points for hand-written solutions that are difficult to read due to poor handwriting and neatness.)

Remember that you must tag your written problems on Gradescope, or you will receive **no credit** as mentioned in the syllabus.

Groups: This homework is to be completed by yourself. Specific guidelines about collaboration are available on the syllabus, but every student will be submitting their own submission. Please cite any collaboration at the top of your submission.

1. Softball (10 points)

Ten people (7 children and 3 adults) on a softball team show up for a game. (a) How many ways are there to choose 4 players to take the field? (b) How many ways are there to assign the 4 positions by selecting players from the 10 people who show up? (c) How many ways are there to choose 4 players to take the field if at least one of these players must be an adult?

2. Getting from here to there (20 points)

- (a) [4 Points] How many paths are there from point (0,0) to (100,150) if every step increments one coordinate and leaves the other unchanged?
- (b) [4 Points] How many paths are there from point (0,0) to (100,150) if every step increments one coordinate and leaves the other unchanged and you want the path to go through (80,70)?
- (c) [6 Points] How many paths are there from point (0,0) to (100,150) if every step increments one coordinate and leaves the other unchanged and the path can *not* go through (40, 50) or (80,70)? (Try inclusion-exclusion.)
- (d) [6 Points] How many paths are there from point (0,0,0) to (100,40,80) if every step increments one coordinate and leaves the other two unchanged?

3. 5 card hands (10 points)

How many ways are there to select 5 cards from a standard deck of 52 cards if we require that all 4 suits are represented? Order doesn't matter.

4. Arrangements (15 points)

How many ways are there to arrange the letters in the following words? (a) goooogle; (b) possessionlessness.

5. Sitting around (15 points)

Archer (A), Bilbo (B), Cersei (C), Dante (D), Eowyn (E), Frodo (F), and Gollum (G) are sitting in a row of nine seats (Note: there are only seven people). Archer and Bilbo are enemies, so they cannot sit next to each other. Cersei and Dante are dating, so they must sit next to each other. Eowyn, Frodo, and Gollum are best friends, so they also want to sit next to each other, but Frodo must be in the middle of Eowyn and Gollum (with no spaces between the three). Our goal is to figure out how many ways they can sit in a row. Build up to the answer by answering the following questions: In how many ways can they sit in a row? (Hint: We will start by grouping Eowyn, Frodo, and Gollum, as well as Cersei and Dante. Then we will work on placing Archer and Bilbo). Also, you will find the problem easier if you just call them, A,B,C,D, E, F, and G, as we do henceforth.

- [3 Points] How many ways there are to place the 7 people into the 9 chairs if EFG must sit together in that order and CD must sit together in that order (This is not unlike the rearrangements of DOGGY that we discussed/will discuss in lecture (where the empty seats are like the two Gs.)
- [4 Points] How many ways there are to place the 7 people into the 9 chairs if EFG must sit together but E and G can swap positions and CD must sit together in either order?
- [4 Points] How many ways there are to place the 7 people into the 9 chairs if EFG must sit together (but E and G can swap positions), CD must sit together in either order and AB must sit together in either order?
- [4 Points] How many ways there are to place the 7 people into the 9 chairs if EFG must sit together (but E and G can swap positions), CD must sit together in either order and A and B must not sit next to each other?

6. Binomial Theorem applications (15 points)

- [7 Points] What is the coefficient of x^5y^{12} in the expansion of $(2x - y^2)^{11}$?
- [8 Points] Use the binomial theorem to prove that

$$\sum_{i=0}^{200} \binom{200}{i} (-4)^{200-i} = 3^{200}$$

7. Coding (15 points)

- [10 Points] Read the [Pset1 Coding](#) lesson on Edstem and follow the directions to complete 5 coding exercises. Then submit all required files to PSet1 [Coding] on Gradescope. The score that appears on Gradescope for this part is final.
- [5 Points] Read the [Edstem lesson](#) on Python's numpy library, after completing the previous part. You do **not** need to complete any coding exercises or submit anything to Gradescope for this part. The exercise that is there is entirely OPTIONAL, and intended only for practice if you need it. Afterward, write below what you felt was the most confusing numpy function and/or class to you and why. If nothing is confusing, explain which function and/or class is the most interesting to you. We will grade based on completion and effort rather than correctness, and it's recommended that your answer be no longer than 5 sentences.