$X_i \sim Ber(p) \qquad 1 \le i \le n$

$$E[X_i^2] = 0^2 p_{X_i}(0) + 1^2 p_{X_i}(1)$$
$$= 0 \cdot (1-p) + 1 \cdot p = p$$

$X = \sum_{i=1}^{n} X_i \qquad \left(\sum_{i=1}^{n} X_i\right)^2 \neq \sum_{i=1}^{n} X_i^2$

$$E[X^2] = E\left[\left(\sum_{i=1}^{n} X_i\right)^2\right]$$

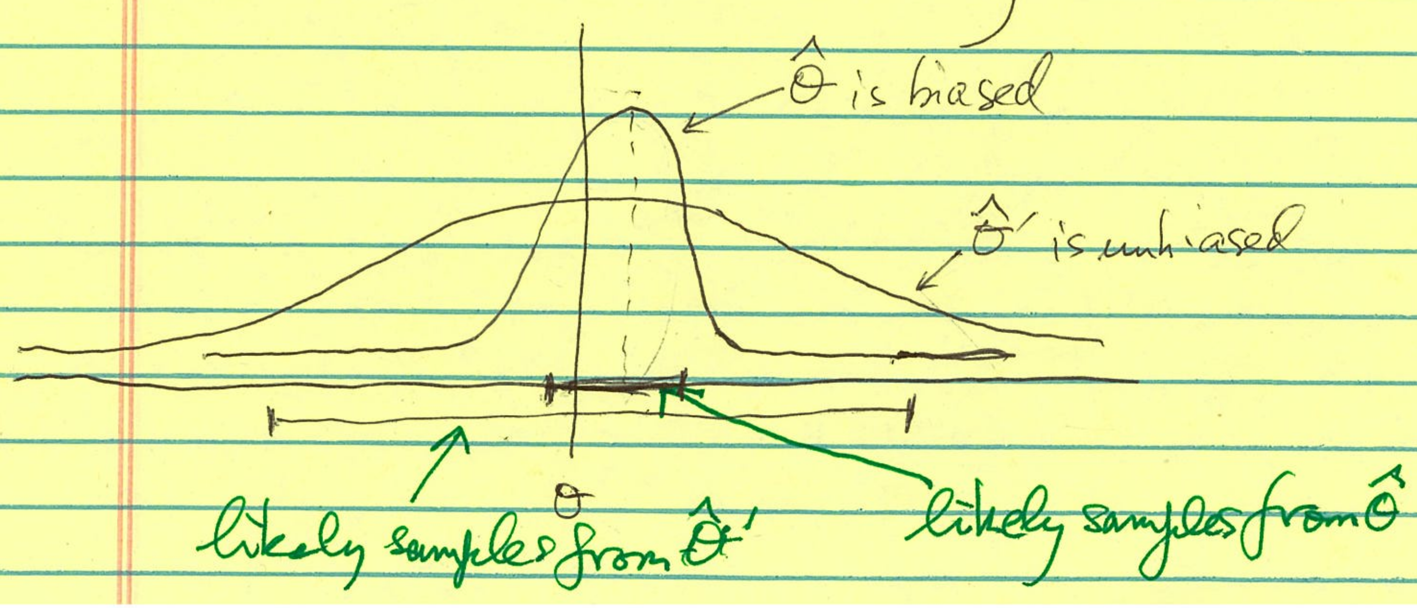
$X \sim N(\theta_1, \theta_2)$

$\hat{\theta}_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{\theta}_1)^2$ is biased, but MLE

$\hat{\theta}_2' = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \hat{\theta}_1)^2$ is unbiased

$$E[\hat{\theta}_2] = \frac{n-1}{n}\theta_2$$

Is an unbiased estimator necessarily best?



MLEs worth memorizing

1. $x_1, x_2, \ldots, x_n \sim Ber(p)$

$$\hat{p} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

2. $x_1, x_2, \ldots, x_n \sim N(\theta_1, \theta_2)$

$$\hat{\theta}_1 = \frac{1}{n}\sum_{i=1}^{n} x_i$$

PMF : discrete r.v. X : $p_X(i) = P(X=i)$

CDF : any r.v. X : $F_X(k) = P(X \le k)$.

PDF : continuous r.v. X : $f_X(x) = \frac{d}{dx} F_X(x)$.

Freivald:

$$P\left(x_j = -\frac{1}{d_j}\Sigma \cdots\right) = P(x_j = S)$$

$$= P(x_j = S \mid S=0)\, \boxed{P(S=0)}$$

$$+ P(x_j = S \mid S=1)\, \boxed{P(S=1)}$$

$$+ P(x_j = S \mid S \notin \{0,1\})\, \boxed{P(S \notin \{0,1\})}$$

$$\cancel{\le \frac{1}{2}\cdot 1 + \frac{1}{2}\cdot 1 + 0\cdot 1}$$

Let $P(S=0) = p$ Then $P(S=1) \le 1-p$

$$P(x_j = S) \le \frac{1}{2}\cdot p + \frac{1}{2}(1-p) + 0(\ )$$

$$= \frac{1}{2}$$