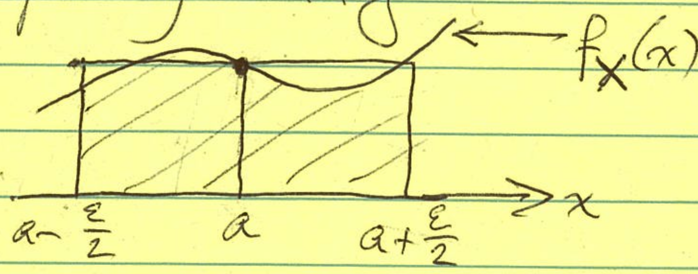
Examples of continuous random variables

1. height of a random person
2. waiting time until an arrival at some server

Exploring density:



$$P\left(a-\frac{\varepsilon}{2} \le X \le a+\frac{\varepsilon}{2}\right) = \int_{a-\frac{\varepsilon}{2}}^{a+\frac{\varepsilon}{2}} f_X(x)\,dx \approx \varepsilon f_X(a)$$

The probability that X is "near" a is proportional to $f_X(a)$.

For a continuous r.v. X, we usually substitute $\int$ for Σ, and f_X for p_X.

$$E[X] = \int_{-\infty}^{+\infty} x f_X(x)\,dx$$

$$E[aX+b] = aE[X]+b$$

$$E[X+Y] = E[X]+E[Y]$$

$$E[g(X)] = \int_{-\infty}^{+\infty} g(x) f_X(x)\,dx$$

$$\mathrm{Var}(X) = E[(X-\mu)^2] = E[X^2]-(E[X])^2$$

X and Y are <u>independent</u> iff $\forall A \forall B$

$$P(X\in A \cap Y\in B) = P(X\in A)P(Y\in B)$$

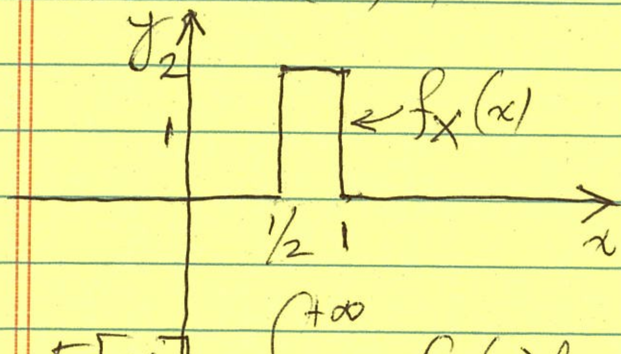
Important Continuous r.v.'s

Uniform

For $\alpha < \beta$, $X \sim Uni(\alpha, \beta)$ iff

$$f_X(x) = \begin{cases} \frac{1}{\beta-\alpha}, & \text{if } x \in [\alpha, \beta] \\ 0, & \text{otherwise} \end{cases}$$

Ex: $X \sim Uni(\frac{1}{2}, 1)$



$$E[X] = \int_{-\infty}^{+\infty} x f_X(x)\,dx = \int_\alpha^\beta x \cdot \frac{1}{\beta-\alpha}\,dx$$

$$= \frac{1}{\beta-\alpha} \cdot \frac{1}{2}x^2 \Big|_\alpha^\beta = \frac{1}{2}\frac{\beta^2-\alpha^2}{\beta-\alpha} = \frac{1}{2}(\alpha+\beta)$$

Exponential: time until the next event, where events happen independently at a rate of λ per unit time.

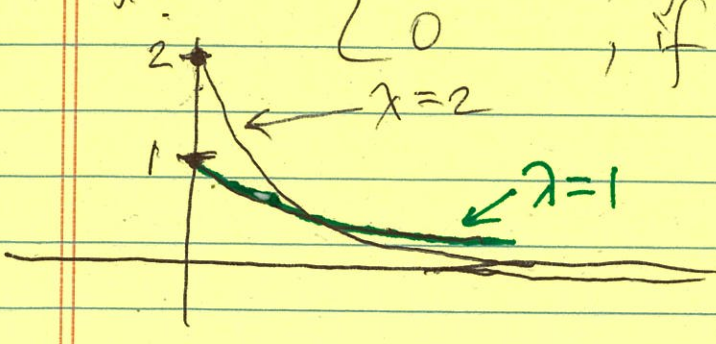
Ex: Radioactive decay: how much time until next α particle is emitted.

$X \sim Exp(\lambda)$

$P(X \geq t) = e^{-\lambda t}$ for any real t

$F_X(t) = P(X \leq t) = 1 - P(X \geq t) = 1 - e^{-\lambda t}$

$$f_X(t) = \begin{cases} \lambda e^{-\lambda t}, & \text{if } t \geq 0 \\ 0, & \text{if } t < 0 \end{cases}$$



$E[X] = \frac{1}{\lambda}$ See this week's worksheet

$Var(X) = \frac{1}{\lambda^2}$

Memorylessness: If $X \sim Exp(\lambda)$, then
$P(X > s+t \mid X > s) = P(X > t)$, for $s, t > 0$.

Poisson: How many events in a given unit of time?

Exp: How much time until the next event?

Geometric: discrete analog of exponential.