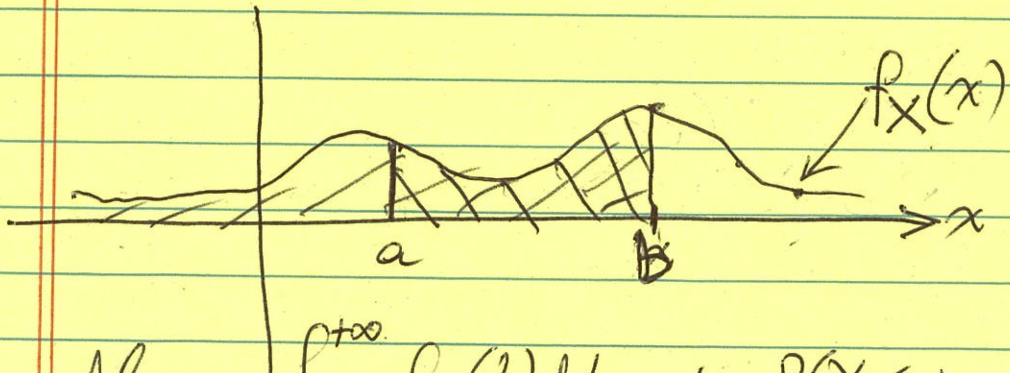
## Continuous Random Variables

Defn: For a random variable $X$, the cumulative distribution function (CDF) is defined as
$F_X(x) = P(X \le x)$.
Notice that $F_X$ is monotonic nondecreasing, because $x < y \Rightarrow F_X(x) \le F_X(y)$.

Defn: A continuous random variable $X$ is one for which $F_X : \mathbb{R} \to \mathbb{R}$ is continuous everywhere.
Notice that a continuous r.v. takes on uncountably infinitely many values.

Defn: Let $X$ be a continuous random variable. The probability density function (pdf) or density $f_X : \mathbb{R} \to \mathbb{R}$ is defined as $f_X(x) = \frac{d}{dx} F_X(x)$.

Turning this around, $F_X(x) = P(X \le x) = \int_{-\infty}^{x} f_X(t)\,dt$
From this, $P(a \le X \le b) = F_X(b) - F_X(a) = \int_a^b f_X(t)\,dt$.




Also, $\int_{-\infty}^{+\infty} f_X(t)\,dt = 1 = P(X \le +\infty)$ and $f_X(t) \ge 0$, for all $t \in \mathbb{R}$.

$$P(X=a) = P(a \le X \le a) = F_X(a) - F_X(a) = 0$$

Densities are not probabilities. In fact, $f_X(a)$ may be $>1$ for some values of $a$.