Some important discrete random variables

A. X is uniform on $[a,b]$, where a and b are integers, denoted $X \sim Unif(a,b)$, if X is equally probable to be any integer in $[a,b]$.

Ex: one roll of a fair die in $Unif(1,6)$.

$p_X(i) = \frac{1}{b-a+1}$ for all ~~$a \le i \le b$~~ $i \in \{a, a+1, \cdots, b\}$

$E[X] = \frac{1}{2}(a+b)$, $Var(X) = \frac{1}{12}(b-a)(b-a+2)$.

B. A Bernoulli r.v., denoted $X \sim Ber(p)$, is an indicator r.v.

$P(X=1) = p, \quad P(X=0) = 1-p$

Ex: one flip of a coin that has probability p of heads.

$E[X] = P(X=1) = p$

$E[X^2] = 1^2 \cdot p + 0^2(1-p) = p$

$Var(X) = E[X^2] - (E[X])^2 = p - p^2 = p(1-p)$.

C. A binomial r.v., denoted $X \sim Bin(n,p)$, is the sum of n independent $Ber(p)$ r.v.'s.

Ex: number of heads in n independent flips of a coin with probability p of heads.

$E[X] = np$

Let $X_1, X_2, \cdots, X_n$ be $Ber(p)$ and independent.

$X = \sum_{i=1}^{n} X_i$

$Var(X) = Var\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} Var(X_i)$ (because independent)

$= \sum_{i=1}^{n} p(1-p) = np(1-p)$

~~E. Algeometric~~ $p_X(i) = P(X=i) = \binom{n}{i} p^i (1-p)^{n-i}$

D. A geometric r.v., denoted $X \sim geo(p)$, is the number of iid flips of a coin with probability p of heads up to and including the first head.

$P_X(i) = (1-p)^{i-1} p$

$E[X] = \frac{1}{p}$.