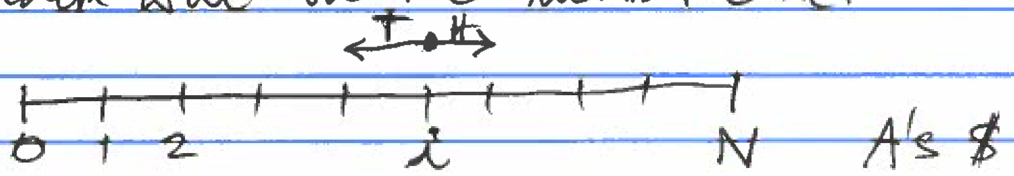
Gambler's Ruin

A has \$$i$ and B has \$$(N-i)$

Flip a fair coin: $H \Rightarrow$ A wins \$1 from B,
$T \Rightarrow$ B wins \$1 from A.

Whoever got all \$$N$ wins.

Random walk on the number line.



Let E_i be the event that A wins the game, starting with \$$i$. What is $P(E_i)$?

Condition on the first flip and use LTP.

$$p_i = P(E_i) = P(E_i \mid H)P(H) + P(E_i \mid T)P(T) \quad \text{(LTP)}$$

$$= P(E_{i+1})\cdot\tfrac{1}{2} + P(E_{i-1})\cdot\tfrac{1}{2}$$

$$= \tfrac{1}{2}(p_{i+1} + p_{i-1})$$

$$2p_i = p_{i+1} + p_{i-1}$$

$$p_i - p_{i-1} = p_{i+1} - p_i$$

$i=1$: $p_2 - p_1 = p_1 - p_0 = p_1 - 0 = p_1$

$p_2 = 2p_1$, $p_3 = p_2 + p_1 = 3p_1$, $p_4 = p_3 + p_1 = 4p_1$, ...

$p_i = i p_1$

$p_N = N p_1 = 1$ so $p_1 = \frac{1}{N}$

$p_i = \frac{i}{N}$ for every $0 \le i \le N$

If you gamble at a casino, $i = 1000$ and $N = 10{,}000{,}000$