Equally Likely Outcomes (cont.)

Ex: 2 defective chips are manufactured, 2 of them defective. You pick a sample of $k \leq n - d$ chips randomly for testing, with each set of k equally probable. What is $P(\geq 1$ defective chip in the sample of k)?

Let $E$ be the event that the sample has $0$ defective chips. Let $\Omega$ be the set of all possible samples of size $k$.

$$P(E) = \frac{\binom{n-d}{k}}{\binom{n}{k}}$$

$$P(\geq 1\text{ defective chip}) = P(\bar{E}) = 1 - P(E) = 1 - \frac{\binom{n-d}{k}}{\binom{n}{k}}$$

Conditional probability of $E$ given $F$, written $P(E|F)$, where $F \neq \phi$ is the probability that $E$ occurs, given that $F$ was observed.

Sample space is reduced to $F$, and the event is reduced to $E \cap F$.

With equally likely outcomes,

$$P(E|F) = \frac{|E|\cap |F|}{|F|}$$

$$= \frac{|E\cap F|}{|F|/|\Omega|} = \frac{P(E\cap F)}{P(F)}$$

Definition of cond prob. in the general case.
Ex. Roll a fair die. What is \( P(5 \mid \text{odd}) \)?

\[
E = 1, 2, 3, 4, 5, 6 \quad F = 3, 1, 3, 5.
\]

1. From counting, \( P(E \mid F) = \frac{1}{\text{\#E}} = \frac{1}{\text{\#F}} = \frac{1}{3} \)

2. From prob., \( P(E \mid F) = \frac{P(E \mid F) \cdot P(F)}{P(F)} = \frac{1/6}{1/2} = \frac{1}{3} \)

Ex. Assume 2J is face up. Let \( A \) be all ways of dealing 2 Schnapsen hands. Let \( Y \) be the event that you have 3 trumps, and \( O \) be the event that your opponent has 0 trumps.

\[
P(O \mid Y) = \frac{10 \times 9}{10} = \frac{90}{10} = \frac{90}{10} = 10.9.8.7.6
\]

Compare to the unconditional probability of being dealt no trumps, \( \approx 0.258 \)

Outcomes not necessarily equally likely.

Ex. Let \( X = \text{no. of heads if I flip a fair coin twice}, P(0) = P(2) = \frac{1}{4}, P(1) = \frac{1}{2} \).

General definition of conditional probability is

\[
P(E \mid F) = \frac{P(E \cap F)}{P(F)}
\]

Ex. Let \( E \) be the event "2 heads" and let \( F \) be the event "at least 1 head".

\[
P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{P(E)}{P(F)} = \frac{1/4}{1 - 1/4} = \frac{1/4}{3/4} = \frac{1}{3}
\]

Chain rule: \( P(E \cap F) = P(E \mid F) \cdot P(F) \)