Equally Likely Outcomes (cont.)

Ex: Quality control

n chips are manufactured, d of them defective. You pick a sample of $k \le n-d$ chips randomly for testing, with each set of k equally probable. What is $P(\ge 1 \text{ defective chip in the sample of } k)$?

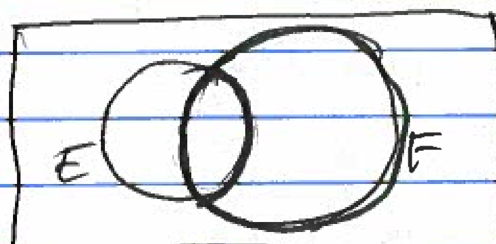
Let E be the event that the sample has 0 defective chips. Let Ω be the set of all possible samples of size k.

$$P(E) = \frac{\binom{n-d}{k}}{\binom{n}{k}}$$

$$P(\ge 1 \text{ defective chip}) = P(\bar{E}) = 1 - P(E) = 1 - \frac{\binom{n-d}{k}}{\binom{n}{k}}.$$

Conditional probability of E given F, written $P(E|F)$, where $F \ne \emptyset$ is the probability that E occurs, given that F was observed.

Sample space is reduced to F, and the event is reduced to $E \cap F$



With equally likely outcomes,

$$P(E|F) = \frac{|E \cap F|}{|F|}. \quad \text{(useful when equally likely)}$$

$$= \frac{|E \cap F| / |\Omega|}{|F| / |\Omega|} = \boxed{\frac{P(E \cap F)}{P(F)}} \quad \text{Definition of cond prob. in the general case.}$$

Ex: Roll a fair die, what is $P(5 \mid odd)$?

$E = \{5\}$, $F = \{1,3,5\}$.

1. From counting, $P(E|F) = \frac{|E \cap F|}{|F|} = \frac{|E|}{|F|} = \frac{1}{3}$
2. From prob., $P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{P(E)}{P(F)} = \frac{1/6}{1/2} = \frac{1}{3}$

Ex: Assume ♣J is face up. Let Ω be all ways of dealing 2 Schnapsen hands. Let Y be the event that you have 0 trumps, and Θ be the event that your opponent has 0 trumps.

$$P(\Theta|Y) = \frac{|\Theta \cap Y|}{|Y|} = \frac{\cancel{\binom{15}{5}}\binom{10}{5}}{\cancel{\binom{15}{5}}\binom{14}{5}} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6}{14\cdot 13\cdot 12\cdot 11\cdot 10} \approx 0.126$$

Compare to the unconditional probability of being dealt no trumps, ≈ 0.258.

Outcomes not necessarily equally likely.

Ex: Let $\Omega = \{0,1,2\}$, the number of heads if I flip a fair coin twice. $P(0) = P(2) = \frac{1}{4}$, $P(1) = \frac{1}{2}$.

General definition of conditional probability is

$$P(E|F) = \frac{P(E\cap F)}{P(F)}$$

Ex: Let E be the event "2 heads" and let F be the event "at least 1 head".

$$P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{P(E)}{P(F)} = \frac{1/4}{1-1/4} = \frac{1/4}{3/4} = \frac{1}{3}$$

Chain rule: $P(E\cap F) = P(E|F)P(F)$