Implications of the axioms:

Defn: If Ω is the sample space and $E \subseteq \Omega$, then $\bar{E} = \Omega - E$.

(a) $P(\bar{E}) = 1 - P(E)$, because

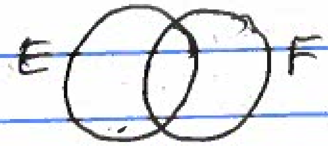
$$1 = P(\Omega) = P(E \cup \bar{E}) = P(E) + P(\bar{E})$$

(b) If $E \subseteq F$, then $P(E) \le P(F)$, because

$$P(F) = P(E \cup (F-E)) = P(E) + P(F-E)$$
$$\ge P(E) + 0 = P(E)$$

(c) $P(E) \le 1$, because $E \subseteq \Omega$, so $P(E) \le P(\Omega) = 1$

(d) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$: inclusion-exclusion



$$E \cup F = (E-F) \cup (F-E) \cup (E \cap F)$$

Apply axiom 3 to this RHS.

Equally Likely Outcomes:

Ω is finite in this case.

If $a \in \Omega$, then $P(a) = \frac{1}{|\Omega|}$, for any a. $P(\{a\})$

Ex: Flip of a fair coin.

2 flips of a fair coin

Roll of a fair die

Suppose $E \subseteq \Omega$.

$$P(E) = P\left(\bigcup_{a \in E} \{a\}\right) = \sum_{a \in E} P(a) = \sum_{a \in E} \frac{1}{|\Omega|} = \frac{|E|}{|\Omega|}$$

Ex: Assume ♠J is face-up. Assume that any 5-card hand is equally likely.

$$P(\text{no trump in initial hand}) = \frac{|E|}{|\Omega|} = \frac{\binom{15}{5}}{\binom{19}{5}}$$

$$= \frac{3003}{11628} \approx 0.258$$

Ex: Assume your 5-card hand is dealt before the trump is turned up.

$$P(\geq 1 \text{ marriage in initial hand}) = \frac{\binom{4}{1}\binom{18}{3} - \binom{4}{2}\binom{16}{1}}{\binom{20}{5}}$$

$$\approx 0.204$$

Note change of Ω from previous example.

$\binom{4}{1}\binom{18}{3}$: why ~~this~~ overcounts:

1. ♠ ♣QK♦A
2. ♣ ♠QK♦A