Overcounted formula $\binom{4}{1}\binom{18}{4} = \boxed{12{,}240}$

How many hands have 0 trumps? $\binom{15}{5}$

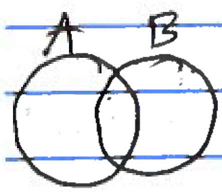
How many possible hands in total? $\binom{19}{5}$

How many have ≥ 1 trump?

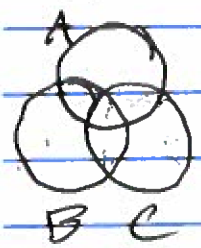
$$\binom{19}{5} - \binom{15}{5} = \frac{19!}{5!\,14!} - \frac{15!}{5!\,10!} =$$

$$\frac{19 \cdot 18 \cdot 17 \cdot 16 \cdot 15}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} - \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \boxed{8{,}625}$$

Inclusion-exclusion:



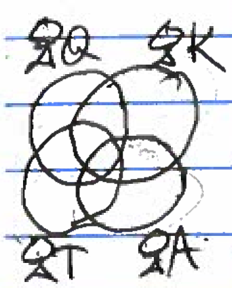
$$|A \cup B| = |A| + |B| - |A \cap B|$$



$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

In general: +singles − doubles + triples − quads + ⋯

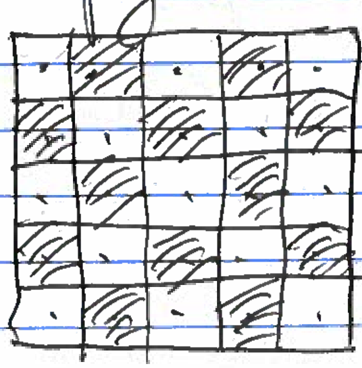
How many Schnapsen hands have ≥ 1 trump, given ♠J is face up?



$$\binom{4}{1}\binom{18}{4} - \binom{4}{2}\binom{17}{3} + \binom{4}{3}\binom{16}{2} - \binom{4}{4}\binom{15}{1} = \boxed{8{,}625}$$

Pigeonhole Principle: If there are n pigeons and k pigeonholes, where $k < n$, then there is some pigeonhole with at least 2 pigeons.

Ex:



5×5 chessboard, one flea on each square. When you ring a bell, every flea jumps to an adjacent square (not diagonally). After the jump, some square has ≥ 2 fleas.

13 white & 12 black squares. The 13 fleas that start on white all end up on black. By pigeonhole principle, some black square has ≥ 2 fleas.

Intro to probability

1. Sample space: set Ω of possible "outcomes" of an experiment.

Ex: coin flip. $\Omega_1 = \{H, T\}$

2 coin flips. $\Omega_2 = \{(H,H), (H,T), (T,H), (T,T)\}$

die roll: $\Omega_3 = \{1,2,3,4,5,6\}$

5-card Schnapsen hands, assuming ♣J is face-up.

$|\Omega_4| = \binom{19}{5}$

2. An event is any $E \subseteq \Omega$.

Ex: ≥ 1 head in 2 coin flips: $E_1 = \{(H,H),(H,T),(T,H)\}$

odd roll of die: $E_2 = \{1,3,5\}$

5-card Schnapsen hands with 0 trumps: $|E_4| = \binom{15}{5}$.

3. Defn: E and F are mutually exclusive (disjoint) iff $E \cap F = \emptyset$.

Axioms of probability: There is a function P that assigns a real number $P(E)$ to every event E satisfying:

(1) $P(E) \geq 0$,

(2) $P(\Omega) = 1$, and

(3) If E and F are mutually exclusive, then $P(E \cup F) = P(E) + P(F)$.