\[ \binom{n}{k} = \frac{n!}{k!(n-k)!} \]
"number of combinations of objects taken k at a time"
"n choose k"
"binomial coefficients"

\[ P(n,k) : \text{number of sequences of } k \text{ objects from } 1, 2, \ldots, n \]
\[ (n)_k : \text{number of sets of } k \text{ objects from } 1, 2, \ldots, n \]

Example:

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          Pike
Union
University
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How many routes from 1st Seneca to 5th Pike that go either N or E at each intersection?

\[ \frac{7!}{3!4!} = \binom{7}{3} = \binom{7}{4} \]

Identities:

1. \( \binom{n}{k} = \binom{n}{n-k} \), where \( 0 \leq k \leq n \).
2. \( \binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k} \), where \( 1 \leq k \leq n-1 \).

Consider the first of the \( n \) objects, say \( x \). Either

(a) \( x \) is in the subset of size \( k \) (\( \binom{n-1}{k-1} \) ways to finish)
(b) \( x \) is not in the subset of size \( k \) (\( \binom{n-1}{k} \) ways to finish).
(3) **Binomial Theorem:** \((x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}\)

Proof by counting:

\((x+y)^n = \overbrace{(x+y)(x+y) \ldots (x+y)}^{n \text{ factors}}\)

**Ex:** \((x+y)^3 = (x+y)(x+y)(x+y)\)

\[= x\times x + x\times y + y\times x + y\times y\]

\[= x^3 + 3x^2y + 3xy^2 + y^3\]

In general, choose either \(x\) or \(y\) from each of the \(n\) factors and multiply these choices; how many of these terms look like \(x^k y^{n-k}\)?

There are \(n\) ways to choose \(x\) in \(k\) of the factors.

\[\sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}\]

**Corollary:** \(\sum_{k=0}^{n} \binom{n}{k} = 2^n\)

**Proof:** \(2^n = (1+1)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} = \sum_{k=0}^{n} \binom{n}{k}\)

**Complementing**

**Ex:** Assume \(\spadesuit\) is face-up trump on the table. How many 5-card Schweppe hands contain 1 trump?

\((\text{1}) \times (\frac{18}{4})\)

No! Overcounting.

- 2\(K\) 2\(\spadesuit\)
- 2\(K\) 2\(\spadesuit\)
- 2\(K\) 2\(\spadesuit\)