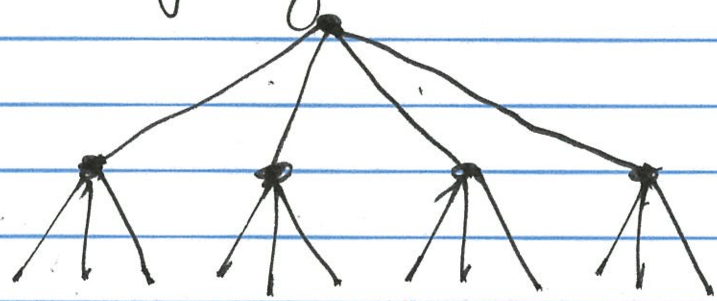
Product Rule: If there are m choices ~~of~~ for step 1 and, for each choice, there are n choices for step 2, then there are mn choices for the two steps together.



Easily generalizes to n steps, by induction.

Ex: How many n-bit strings are there? 2^n

Ex: How many subsets does a set of size n have? 2^n, because, for each element, it is either in or out of the subset.

Ex: How many passwords of length 4 are there, using either lowercase letters or digits? 36^4

Ex: How many if you are not allowed to repeat any character? $36 \cdot 35 \cdot 34 \cdot 33 = 36!/32!$

Permutations: counting when order matters.

How many sequences are there that use 1, 2, 3 each exactly once? $3 \cdot 2 \cdot 1 = 3!$

How many sequences that use $1, 2, 3, \ldots, n$ each exactly once? $n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1 = n!$

Note: $0! = 1$

k-permutations: How many sequences of length k are there using 1, 2, 3, ⋯, n, each used 0 or 1 time? $0 \le k \le n$

$$P(n,k) = n(n-1)(n-2)\cdots(n-k+1) = \frac{n!}{(n-k)!}$$

Passwords: $P(36,4) = \frac{36!}{(36-4)!} = \frac{36!}{32!}$

Combinations: when order doesn't matter.

Ex: Your elf-lord ~~avatar~~ avatar can carry any 3 objects from (1) sword, (2) knife, (3) ~~staff~~, (4) ring, (5) laptop. How many combinations of 3 are there?

$$\binom{5}{3} = \frac{P(5,3)}{3!} = \frac{5!}{3!\,2!} = \frac{5\cdot 4}{2\cdot 1} = 10$$

In general, if $0 \le k \le n$,

$$\binom{n}{k} = \frac{P(n,k)}{k!} = \frac{n!}{k!\,(n-k)!}$$