Quiz Section 0 – CSE 311 Review

The contents of this section are meant to review some recurring concepts from previous classes (in particular, CSE 311).

Task 1 – Sets

a) For each one of the following sets, give its **cardinality**, i.e., indicate how many elements it contains:

- $A = \emptyset$
- $B = \{\emptyset\}$
- $C = \{\{\emptyset\}\}$
- $D = \{\emptyset, \{\emptyset\}\}$

b) Let $S = \{a, b, c\}$ and $T = \{c, d\}$. Compute:

- $S \cup T$
- $S \cap T$
- $S \setminus T$
- 2^S
- T^3

Task 2 – Functions

For two sets A, B, a **function** $f : A \to B$ assigns each $a \in A$ to a (unique) value $b = f(a) \in B$. Here, A is the **domain**, whereas B is the **codomain** of f. Moreover:

- The **range** of f is $f(A) = \{f(a) \, : \, a \in A\}$.
- The function f is **injective** (or **one-to-one**) if $\forall x, y \in A : x \neq y \Longrightarrow f(x) \neq f(y)$.
- The function f is **surjective** (or **onto**) if $\forall b \in B : \exists a \in A : f(a) = b$.
- The function f is bijective if it is both injective and surjective.

Which of the following functions are injective / surjective / bijective? (Here, let $\mathbb{R}_{\geq 0} = \{x \in \mathbb{R} \, : \, x \geq 0\}$.)

a) $f_1 : \mathbb{R} \to \mathbb{R}$ such that $f_1(x) = x^2$.

b) $f_2 : \mathbb{N} \to \mathbb{N}$ such that $f_2(x) = x^2$.

c) $f_3 : \mathbb{R} \to \mathbb{R}_{\geq 0}$ such that $f_3(x) = x^2$.

d) $f_4 : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ such that $f_4(x) = x^2$.

Task 3 – Induction Proofs

a) Is the following proof by induction (over n) correct? Explain!

> **Claim.** For all real numbers $a > 0$, and all integers $n \geq 0$, we have $a^n = 1$.
> **Base case.** For $n = 0$, we have $a^n = a^0 = 1$.
> **Induction hypothesis.** Assume that $a^m = 1$ for all $0 \leq m \leq n$.
> **Induction step.** Then,
> $$a^{n+1} = \frac{a^n \cdot a^n}{a^{n-1}} = \frac{1 \cdot 1}{1} = 1 \, ,$$
> because $a^n = a^{n-1} = 1$ by the induction hypothesis.

b) Prove by induction that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.