

Quiz Section 1

Review

- 1) **Product rule.** $|A_1 \times A_2 \times \cdots \times A_m| = \underline{\hspace{2cm}}$.
- 2) **Permutations.** The number of ways to re-order n elements is $\underline{\hspace{2cm}}$.
- 3) **k -permutations.** The number of ways to choose a sequence of k distinct elements from a set of n elements is $\underline{\hspace{2cm}}$.
- 4) **Subsets.** The number of ways to choose a k -element subset of a set of n elements is $\underline{\hspace{2cm}}$.
- 5) **Set difference.** Is it always true that $|A \setminus B| = |A| - |B|$?
- 6) **Inclusion-exclusion.** $|A \cup B| = \underline{\hspace{2cm}}$.
- 7) **Inclusion-exclusion.** $|A \cup B \cup C| = \underline{\hspace{2cm}}$.

Task 1 – Basic Counting

- a) Credit-card numbers are made of 15 decimal digits, and a 16th checksum digit (which is uniquely determined by the first 15 digits). How many credit-card numbers are there?
- b) How many divisors does $1440 = 2^5 3^2 5$ have?
- c) How many ways are there to arrange the CSE 312 staff on a line (7 TAs, one professor) for a group picture?
- d) How many ways are there to arrange the CSE 312 staff on a line so that Professor Tessaro is either at the left end or right end of the line?

Task 2 – Counting Functions

- a) How many functions $f : D \rightarrow R$ are there?
- b) How many injective functions $f : D \rightarrow R$ are there?
- c) How many surjective functions $f : D \rightarrow R$ are there if $|R| = 2$?

Task 3 – Combinatorial Proofs

Give combinatorial proofs of the following identities:

- a) $\binom{n}{k} = \binom{n}{n-k}$.
- b) $\sum_{k=0}^n \binom{n}{k} = 2^n$.
- c) Show that $\binom{2n}{n} = \sum_{k=0}^n \binom{n}{k}^2$.

Task 4 – Inclusion/Exclusion

- a) How many strings with letters $\{a, b, c\}$ of length 7 are there which contain each letter at least once?
- b) Alice, Bob, Charlie, Dana, and Eve sit at a round table with five seats. Alice does not want to sit next to Bob, and Eve does not want to sit next to Dana. How many seating arrangements are there?

Here, if one seating arrangement can be obtained from the other one via rotation, they are considered equivalent and not counted separately.