CSE 312: Foundations of Computing II Autumn 2019

Practice Midterm 2

(1) Are the following statements True or False? Provide a short justification for your answer. (5 points each)

- The number of subsets of $[n]$ of size k is $\binom{n}{k}$.
- Let $A_1, A_2, \ldots, A_n$ be sets. Then, $|A_1 \cup A_2 \cup \ldots \cup A_n| \leqslant |A_1| + |A_2| + \ldots + |A_n|$.
- $\binom{11}{2} + \binom{11}{4} + \ldots + \binom{11}{10} = \binom{11}{1} + \binom{11}{3} + \ldots + \binom{11}{9}$.
- Let $\mathcal{A}_1, \ldots, \mathcal{A}_n$ be events in a sample space. Then, $\mathbb{P}(\mathcal{A}_1 \cap \mathcal{A}_2 \cap \ldots \cap \mathcal{A}_n) \leqslant \mathbb{P}(\mathcal{A}_1) \cdot \mathbb{P}(\mathcal{A}_2) \ldots \mathbb{P}(\mathcal{A}_n)$.
- If a coin has probability p of giving heads, then the expected number of tosses before you see heads is $1/p$.
- Suppose X and Y are real valued random variables that are both positive with probability 1. Then $\mathbb{E}(2X + Y) > \mathbb{E}(X) + \mathbb{E}(Y)$.

(2) (10 points) There is a train with 3 carriages that are initially empty. It arrives at a train station where 20 people are waiting to get on. Each person chooses one of the carriages. How many ways are there for people to get on the train so that none of the carriages are empty? You may leave your answer as a simplified expression.

(3) (10 points) Company A supplies 40% of the computers sold and is late 5% of the time. Company B supplies 30% of the computers sold and is late 3% of the time. Company C supplies another 30% and is late 2.5% of the time. A computer arrives late; what is the probability that it came from Company A?

(4) (10 points) Let $\mathcal{A}$ and $\mathcal{B}$ be events such that $\mathbb{P}(\mathcal{A}) = 0.1$, $\mathbb{P}(\mathcal{B}) = 0.5$ and $\mathbb{P}(\mathcal{A} \cup \mathcal{B}) = 0.58$. Are $\mathcal{A}$ and $\mathcal{B}$ independent?

(5) (10 points) A die is rolled twice and each roll is independent of the other. Let X_1 and X_2 be the random variables denoting the values of the first and the second roll, respectively. What is $\mathbb{E}(X_1 \cdot X_2)$?