CSE 312: Foundations of Computing II Autumn 2019

Practice Midterm 1

Note: Here and below, the number of re-arrangements of a word refers to the ways of re-arranging the letters which lead to distinct words.

(1) Are the following statements True or False? Provide a short justification for your answer.

- The number of rearrangements of the word JAZZ is $4!$.
- Let $\mathcal{A}_1, \ldots, \mathcal{A}_n$ be a partition of the sample space, and let $\mathcal{B}$ be an event. Then, $\mathbb{P}(\mathcal{B}) = \mathbb{P}(\mathcal{B}|\mathcal{A}_1) + \mathbb{P}(\mathcal{B}|\mathcal{A}_2) + \ldots + \mathbb{P}(\mathcal{B}|\mathcal{A}_n)$.
- If $\mathcal{A}$ and $\mathcal{B}$ are mutually exclusive events ($\mathcal{A} \cap \mathcal{B} = \varnothing$) such that $\mathbb{P}(\mathcal{A}) \neq 0$ and $\mathbb{P}(\mathcal{B}) \neq 0$, then $\mathcal{A}$ and $\mathcal{B}$ are independent.
- Let $\mathcal{A}$ and $\mathcal{B}$ be events such that $\mathbb{P}(\mathcal{A}) = 0.3$, $\mathbb{P}(\mathcal{B}) = 0.4$ and $\mathbb{P}(\mathcal{A} \cup \mathcal{B}) = 0.58$. Then, $\mathcal{A}$ and $\mathcal{B}$ are independent.
- For any random variables X and Y, $\mathbb{E}(aX + bY + c) = a\mathbb{E}(X) + b\mathbb{E}(Y) + c$.

(2) What are the number of re-arrangements of the word POPULAR in which the letters L and A do not occur next to each other?

(3) In a certain day care class, 25% of the children have grey eyes, 50% of them have blue and the other 25%'s eyes are in other colors. One day they play a game together. In the first run, 62% of the grey eye ones, 78% of the blue eyed ones and 54% of the children with other eye color were selected. Now, if a child is selected randomly from the class, and we know that he/she was not in the first game, what is the probability that the child has blue eyes?

(4) Suppose we throw n balls into n bins with the probability of a ball landing in each of the n bins being equal. Each throw is independent of the other throws. What is the expected number of empty bins?

(5) A fair coin (probability of heads is $1/2$) is first tossed with the left hand and then tossed with the right hand. This is repeated until the left hand toss is a head and the following right hand toss is also a head. Every toss is independent of the other tosses. What is the expected number of tosses?