Practice Midterm 1

Note: Here and below, the number of re-arrangements of a word refers to the ways of re-arranging the letters which lead to distinct words.

(1) Are the following statements True or False? Provide a short justification for your answer.

- The number of rearrangements of the word JAZZ is $4!$.
- Let $A_1, \ldots, A_n$ be a partition of the sample space, and let $B$ be an event. Then, $P(B) = P(B|A_1) + P(B|A_2) + \ldots + P(B|A_n)$.
- If $A$ and $B$ are mutually exclusive events ($A \cap B = \emptyset$) such that $P(A) \neq 0$ and $P(B) \neq 0$, then $A$ and $B$ are independent.
- Let $A$ and $B$ be events such that $P(A) = 0.3$, $P(B) = 0.4$ and $P(A \cup B) = 0.58$. Then, $A$ and $B$ are independent.
- For any random variables $X$ and $Y$, $E(ax + by + c) = aE(X) + bE(Y) + c$.

(2) What are the number of re-arrangements of the word POPULAR in which the letters L and A do not occur next to each other?

(3) In a certain day care class, 25% of the children have grey eyes, 50% of them have blue and the other 25%’s eyes are in other colors. One day they play a game together. In the first run, 62% of the grey eye ones, 78% of the blue eyed ones and 54% of the children with other eye color were selected. Now, if a child is selected randomly from the class, and we know that he/she was not in the first game, what is the probability that the child has blue eyes?

(4) Suppose we throw $n$ balls into $n$ bins with the probability of a ball landing in each of the $n$ bins being equal. Each throw is independent of the other throws. What is the expected number of empty bins?

(5) A fair coin (probability of heads is $1/2$) is first tossed with the left hand and then tossed with the right hand. This is repeated until the left hand toss is a head and the following right hand toss is also a head. Every toss is independent of the other tosses. What is the expected number of tosses?