CSE 312: Foundations of Computing II Autumn 2019

Homework 3

Due: Wednesday, Oct 23, by 11:59pm.
Refer to the instructions on Homework 1

Task 1 – Bad Dice [10 pts]

Alice has a bag with three dice in them. In particular:

- **Die 1** has the numbers 1,2,3,4,5,6 on its sides.
- **Die 2** has the number 1 on four sides, and the number 2 on the remaining two sides.
- **Die 3** has the number 2 on three sides, and the numbers 1,4,5 on each of the remaining three sides.

Alice picks one of the three dice from the bag (each is equally likely), and then throws the die. She shows you the outcome, and the outcome only (i.e, you do not see the other faces of the die).

a) What is the probability that the outcome is 1?

b) Given the outcome is 1, what is the probability she used **Die 1**?

c) Given the outcome is even, what is the probability she used **Die 2**?

d) Consider the event $\mathcal{A}$ that Alice picked **Die 1** and the event $\mathcal{E}$ that the outcome is even. Are $\mathcal{A}$ and $\mathcal{E}$ independent?

Task 2 – Medical Tests [10 pts]

A pharmaceutical company publishes results from a trial of its new test for a certain disorder. The **false negative rate** is small: the test returns a negative result for only 6% of patients with the disorder. The **false positive rate** is also small: the test returns a positive result for only 12% of participants that do not have the disorder. Assume that 0.5% (that is, the fraction 0.005) of the population has the disorder.

a) Model a probability space describing the experiment where a single patient can either have the disease or not, and is subject to the test. Probabilities are assigned according to the above statistical percentages.

Hint: You can use the graph-representation of a sequential process we have seen in class.

b) What is the probability of having the disease, given the result of the test is negative?

c) What is the probability of having the deisaese, given the result of the test is positive?

d) Repeat **a)** and **b)** assuming now that 12% of the population has the disease.

Task 3 – Genetics and Blood Type [10 pts]

As you may remember from basic biology, the human A/B/O blood type system is controlled by one gene for which 3 variants ("alleles") are common in the human population – unsurprisingly called A, B, and O. As with most genes, everyone has 2 copies of this gene, one inherited from the mother and the other from the father, and everyone passes a randomly selected copy to each of their children (probability 1/2 for each copy, independently for each child). Focusing only on A and O, people with AA or AO gene pairs have type A blood; those with OO have type O blood. (A is "dominant", O is "recessive".) Suppose Apple and both of her parents have type A blood, but her sister Olive has type O. Give exact answers as simplified fractions.

a) What is the probability that Apple carries an O gene?

b) Apple marries a man with type O blood. What is the probability that their first child will have type O blood?

c) If their first child had type A blood, what is the probability that Apple carries an O gene?

d) If their first child had type A blood, what is the probability that their second child will as well?

Task 4 – Independence [10 pts]

We consider an experiment where three binary values $b_1, b_2, b_3 \in \{0, 1\}$ are chosen randomly, and all 8 outcomes are equally likely. Here, we denote by $\oplus$ the exclusive-or of two bits.

Consider the following four events:

- $\mathcal{A}_1$ is the event that $b_1 \oplus b_2 = 0$
- $\mathcal{A}_2$ is the event that $b_2 \oplus b_3 = 0$
- $\mathcal{A}_3$ is the event that $b_1 \oplus b_3 = 0$
- $\mathcal{A}_4$ is the event that $b_1 = 0$

a) Are the events $\mathcal{A}_1$ and $\mathcal{A}_2$ independent?

b) Are the events $\mathcal{A}_1$, $\mathcal{A}_2$, and $\mathcal{A}_3$ independent?

c) Are the events $\mathcal{A}_1$, $\mathcal{A}_2$, and $\mathcal{A}_4$ independent?

d) Are the events $\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{A}_3$, and $\mathcal{A}_4$ pairwise independent?