CSE 312: Foundations of Computing II
Quiz Section #1: Counting

Review: Main Theorems and Concepts

1. **Product Rule:** Suppose there are m_1 possible outcomes for event A_1, then m_2 possible outcomes for event $A_2, \ldots, m_n$ possible outcomes for event A_n. Then there are ________________________ possible outcomes overall.

2. **Number of ways to order n distinct objects:** ________________________

3. **Number of ways to select from n distinct objects:**

   (a) **Permutations** (number of ways to linearly arrange k objects out of n distinct objects, when the order of the k objects matters): ________________________

   (b) **Combinations** (number of ways to choose k objects out of n distinct objects, when the order of the k objects does not matter): ________________________

4. **Multinomial coefficients:** Suppose there are n objects, but only k are distinct, with $k \leq n$. (For example, "godoggy" has $n = 7$ objects (characters) but only $k = 4$ are distinct: (g, o, d, y)). Let n_i be the number of times object i appears, for $i \in \{1, 2, \ldots, k\}$. (For example, $(3, 2, 1, 1)$, continuing the "godoggy" example.) The number of distinct ways to arrange the n objects is: ________________________

Exercises

Several exercises below deal with a "standard" 52-card deck, such as is used in the games of bridge and poker. This deck consists of 52 cards divided into 4 suits of 13 cards each. The 4 suits are (black) spades ♠, (red) hearts ♡, (black) clubs ♣, and (red) diamonds ♢. The 13 cards ("ranks") of each suit are 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A.

1. How many ways are there to select 5 cards from a standard deck of 52 cards, where the 5 cards contain cards from at most two suits, if:

   (a) order does not matter

   (b) order matters

2. Consider a set of 25 people that form a social network. (The structure of the social network is determined by which pairs of people in the group are friends.) How many possibilities are there for the structure of this social network?

3. Suppose we have 3 diamonds and 3 hearts from a standard deck. How many ways are there to arrange the cards if they have to alternate suit?

4. How many ways are there to choose three initials that have two being the same or all three being the same?

5. A license plate has the form AXYZBCD, where A, B, C, and D are digits and X, Y, and Z are upper case letters. What is the number of different license plates that can be created?

6. A chef is preparing desserts for the week, starting on a Sunday. On each day, only one of five desserts (apple pie, cherry pie, strawberry pie, pineapple pie, and cake) may be served. On Thursday there is a birthday, so cake must be served that day. On no two consecutive days can the chef serve the same dessert. How many dessert combinations are there for the week?

7. A store has 4 books, 14 movies, 6 toys, and 5 posters. In how many ways can a customer buy exactly 1 item from each of exactly 3 categories?

8. In Schnapsen, assuming the stock is not closed, no one has exchanged the jack of trumps, and no marriage has been declared, how many possible orderings of the cards face-down in the stock are there, given the cards you have seen …

   (a) … before trick 1?

   (b) … before trick 2?

   (c) … before trick 3?

   (d) … before trick 4?

   (e) … before trick 5?

9. In how many different ways can you arrange seven people around a circular table?

10. Suppose that 8 people, including you and a friend, line up for a picture. In how many ways can the photographer organize the line if she wants to have fewer than 2 people between you and your friend?

11. Your CSE 312 teaching staff lines up for a picture. How many possible arrangements are there with Maestro Tompa not at either end of the line?

12. How many ways are there to permute the 8 letters A, B, C, D, E, F, G, H so that A is not at the beginning and H is not at the end?

13. There are 40 seats and 40 students in a classroom. Suppose that the front row contains 10 seats, and there are 5 students who must sit in the front row in order to see the board clearly. How many seating arrangements are possible with this restriction?

14. Permutations of objects, some of which are indistinguishable.

    (a) How many permutations are there of the letters in DAWGY?

    (b) How many permutations are there of the letters in DOGGY?

    (c) How many permutations are there of the letters in GODOGGY?

15. A bridge hand consists of 13 cards dealt from a shuffled standard deck of 52 cards. Given a bridge hand consisting of 5 spades, 2 hearts, 3 diamonds, and 3 clubs, in how many ways can the hand be arranged so that the cards of each suit are together ...

    (a) ... but not necessarily sorted by rank within each suit?

    (b) ... and each suit is sorted in ascending rank order?

    (c) ... and each suit is sorted in ascending rank order and the suits are arranged so that the suit colors alternate?

16. Suppose two cards are drawn in order from a bridge deck. In how many ways can the first card be a diamond and the second card a jack?

17. Rabbits Peter and Pauline have three offspring: Flopsie, Mopsie, and Cotton-tail. These five rabbits are to be distributed to four different pet stores so that no store gets both a parent and a child. It is not required that every store gets a rabbit. In how many different ways can this be done?

18. You are playing a game of Schnapsen against the Maestro. The cards you have not seen yet during the current deal are the following:

    ♠ TKJ
    ♡ ATQJ
    ♣ AT
    ♢ KQJ

    Of the possible 5-card hands the Maestro could be holding, how many of them contain at least 18 trick points? Try to find the simplest way to solve this exercise.

19. You have 12 red beads, 16 green beads, and 20 blue beads. How many distinguishable ways are there to place the beads on a string, assuming that beads of the same color are indistinguishable? (The string has a loose end and a tied end, so that reversing the order of the beads gives a different arrangement, unless the pattern of colors happens to form a palindrome.) Try solving the problem two different ways, once using permutations and once using using combinations.

20. There are 12 points on a plane. Five of them are collinear and, other than these, no three are collinear.

    (a) How many lines, each containing at least 2 of the 12 points, can be formed?

    (b) How many triangles, each containing at least 3 of the 12 points, can be formed?

21. You have a triangular prism with top and bottom both being congruent equilateral triangles and the three sides being congruent rectangles. If you pick 5 out of 7 different colors, one to paint each of the 5 faces, how many differently painted triangular prisms can you get? Just rotating the prism does not constitute a different color scheme.

22. There are 6 men and 7 women in a ballroom dancing class. If 4 men and 4 women are chosen and paired off, how many pairings are possible?

23. How many ways are there to seat 10 people, consisting of 5 couples, in a row of 10 seats if . . .

    (a) . . . the seats are assigned arbitrarily?

    (b) . . . all couples are to get adjacent seats?

    (c) . . . the seats are assigned arbitrarily, except that one couple insists on not sitting in adjacent seats?

24. How many bridge hands have a suit distribution of 5, 5, 2, 1? (That is, you are playing with a standard 52-card deck and you have 5 cards of one suit, 5 cards of another suit, 2 of another suit, and 1 of the last suit.)

25. A hand in "draw poker" consists of 5 cards dealt from a shuffled 52-card standard deck.

    (a) How many different hands are there that form a flush? (A hand is said to form a flush if all 5 cards are from the same suit.)

    (b) How many different hands are there that form a straight? (A hand is said to form a straight if the ranks of all 5 cards form an incrementing sequence. The suits do not matter. The lowest straight is A, 2, 3, 4, 5 and the highest straight is 10, J, Q, K, A.)

(c) How many different hands are there that form one pair? (This occurs when the cards have ranks a, a, b, c, d, where a, b, c, and d are all distinct. The suits do not matter.)

(d) How many different hands are there that form two pairs? (This occurs when the cards have ranks a, a, b, b, c, where a, b, and c are all distinct. The suits do not matter.)

(e) How many different hands are there that form three of a kind? (This occurs when the cards have ranks a, a, a, b, c, where a, b, and c are all distinct. The suits do not matter.)

(f) How many different hands are there that form a full house? (This occurs when the cards have ranks a, a, a, b, b, where a and b are distinct. The suits do not matter.)

(g) How many different hands are there that form four of a kind? (This occurs when the cards have ranks a, a, a, a, b. The suits do not matter.)

(h) How many different hands are there that form a straight flush? (This occurs when the cards form a straight and a flush; i.e., a straight with all 5 cards of the same suit)