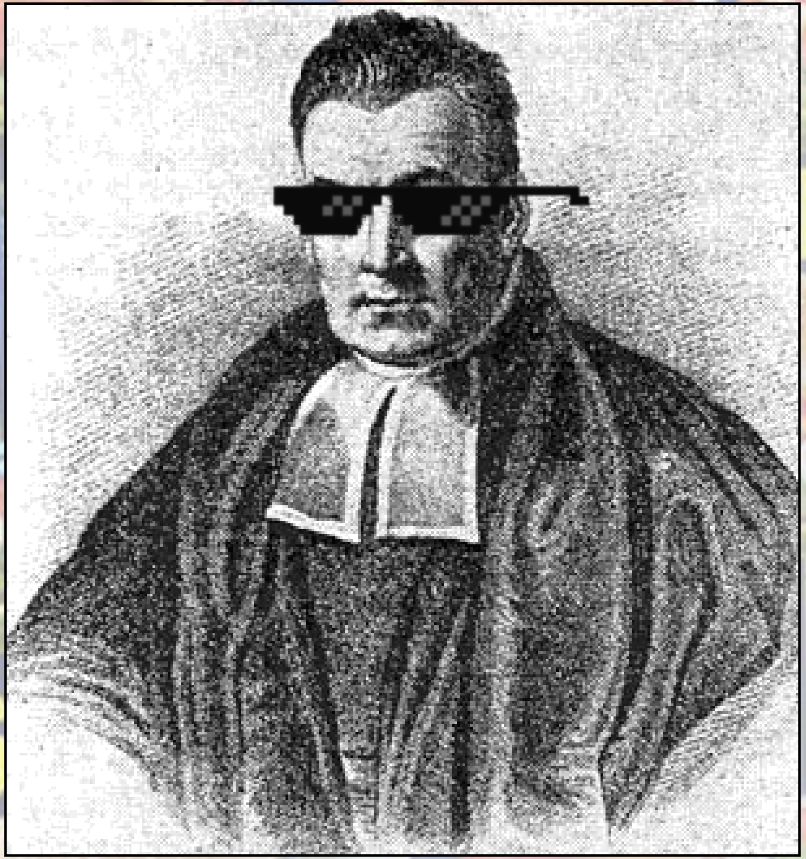
Naïve Bayes Classifiers



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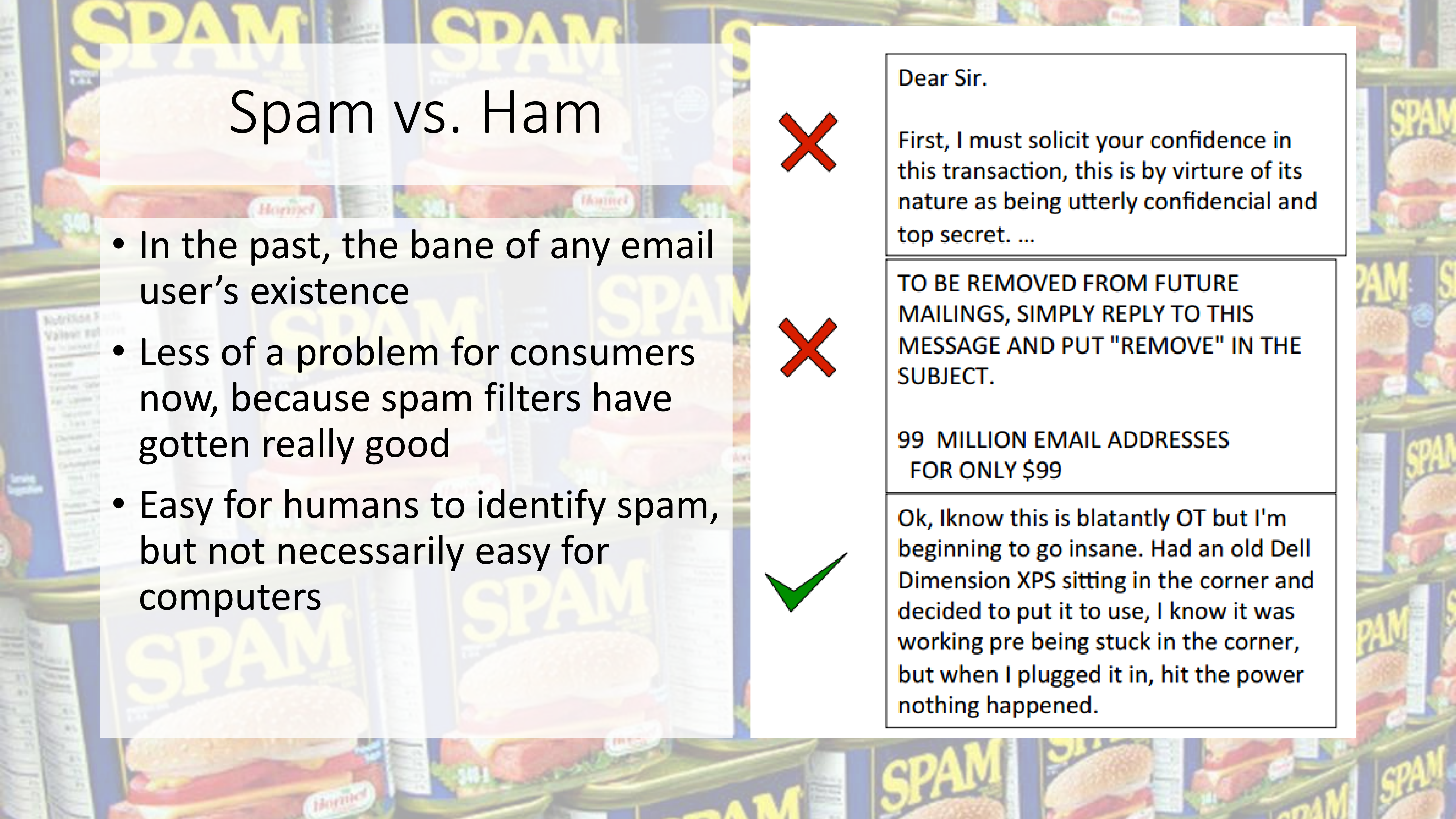
Programming Project: Spam Filter

- Implement a Naive Bayes classifier for classifying emails as either spam or ham (= nonspam).
- You may use Java or Python 3. We’ve provided starter code in both.
- Read Jonathan Lee’s notes on the course web, start early, and ask for help if you get stuck!



Spam vs. Ham

- In the past, the bane of any email user's existence
- Less of a problem for consumers now, because spam filters have gotten really good
- Easy for humans to identify spam, but not necessarily easy for computers

✗

Dear Sir.

First, I must solicit your confidence in this transaction, this is by virture of its nature as being utterly confidencial and top secret. …

✗

TO BE REMOVED FROM FUTURE MAILINGS, SIMPLY REPLY TO THIS MESSAGE AND PUT "REMOVE" IN THE SUBJECT.

99 MILLION EMAIL ADDRESSES
FOR ONLY $99

✓

Ok, Iknow this is blatantly OT but I'm beginning to go insane. Had an old Dell Dimension XPS sitting in the corner and decided to put it to use, I know it was working pre being stuck in the corner, but when I plugged it in, hit the power nothing happened.

The spam classification problem

- Input: collection of emails, already labeled *spam* or *ham*
  - Someone has to label these by hand
  - Called the *training data*
- Use this data to train a model that “understands” what makes an email spam or ham
  - We’re using a Naïve Bayes classifier, but there are other approaches
  - This is a *Machine Learning* problem (take CSE 446 for more)
- Test your model on emails whose label isn’t provided, and see how well it does
  - Called the *test data*

Naïve Bayes in the real world

- One of the oldest, simplest methods for *classification*
- Powerful and still used in the real world/industry
  - Identifying credit card fraud
  - Identifying fake Amazon reviews
  - Identifying vandalism on Wikipedia
  - **Still** used (with modifications) by Gmail to prevent spam
  - Facial recognition
  - Categorizing Google News articles
  - Even used for medical diagnosis!

Naïve Bayes in theory

We will use what we've learned in the past few weeks. Specifically:

- Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- Bayes' Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

- Law of Total Probability

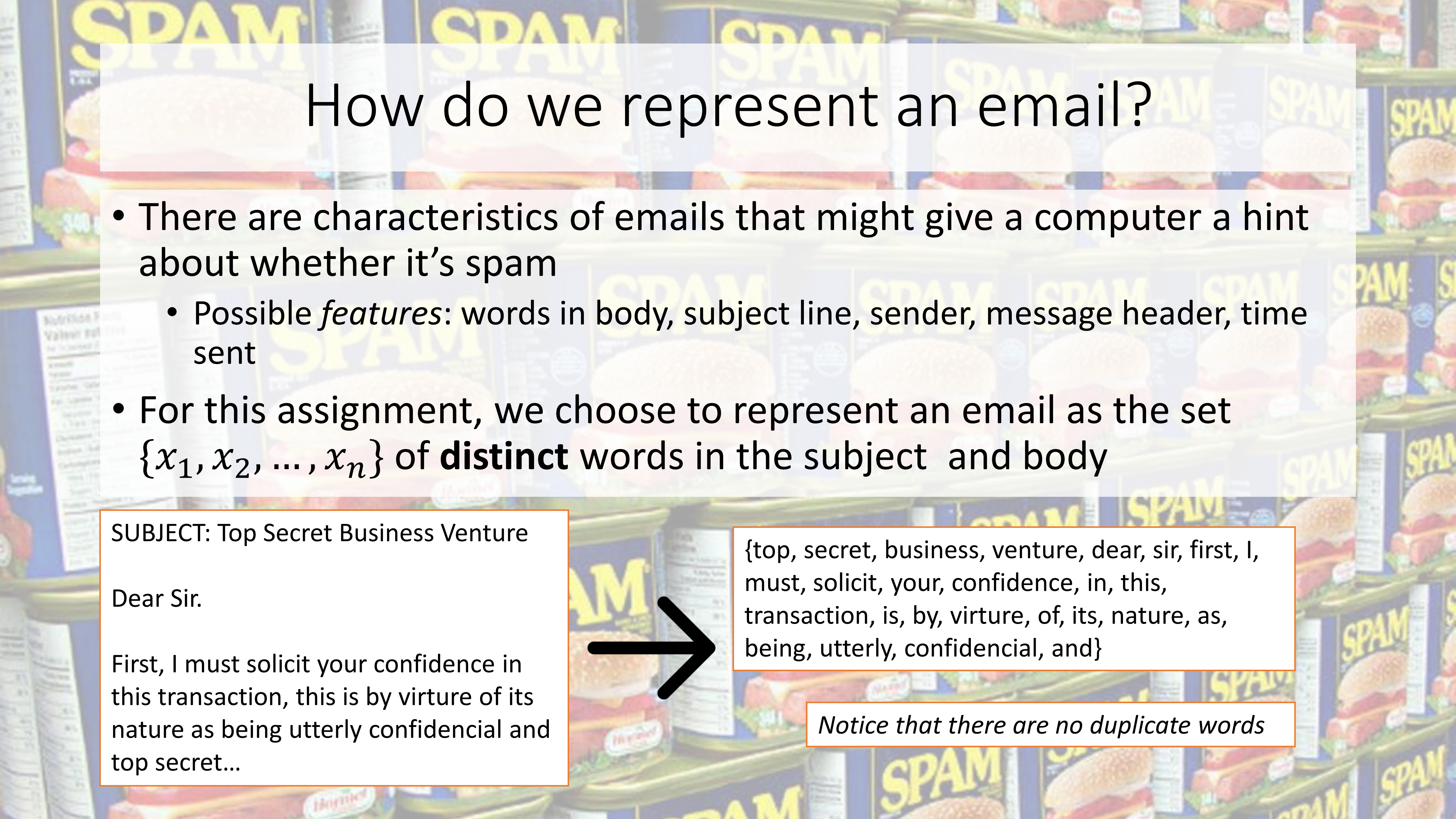
$$P(A) = \sum_n P(A|B_n)P(B_n)$$

- Chain Rule

$$P(A_1, \ldots, A_n)$$
$$= P(A_1)\, P(A_2|A_1) \ldots P(A_n|A_{n-1} \ldots A_1)$$

- Conditional Independence of *A* and *B*, given *C*

$$P(A \cap B|C) = P(A|C)P(B|C)$$
$$P(A|B \cap C) = P(A|C)$$



How do we represent an email?

- There are characteristics of emails that might give a computer a hint about whether it's spam
  - Possible *features*: words in body, subject line, sender, message header, time sent
- For this assignment, we choose to represent an email as the set $\{x_1, x_2, \dots, x_n\}$ of **distinct** words in the subject and body

SUBJECT: Top Secret Business Venture

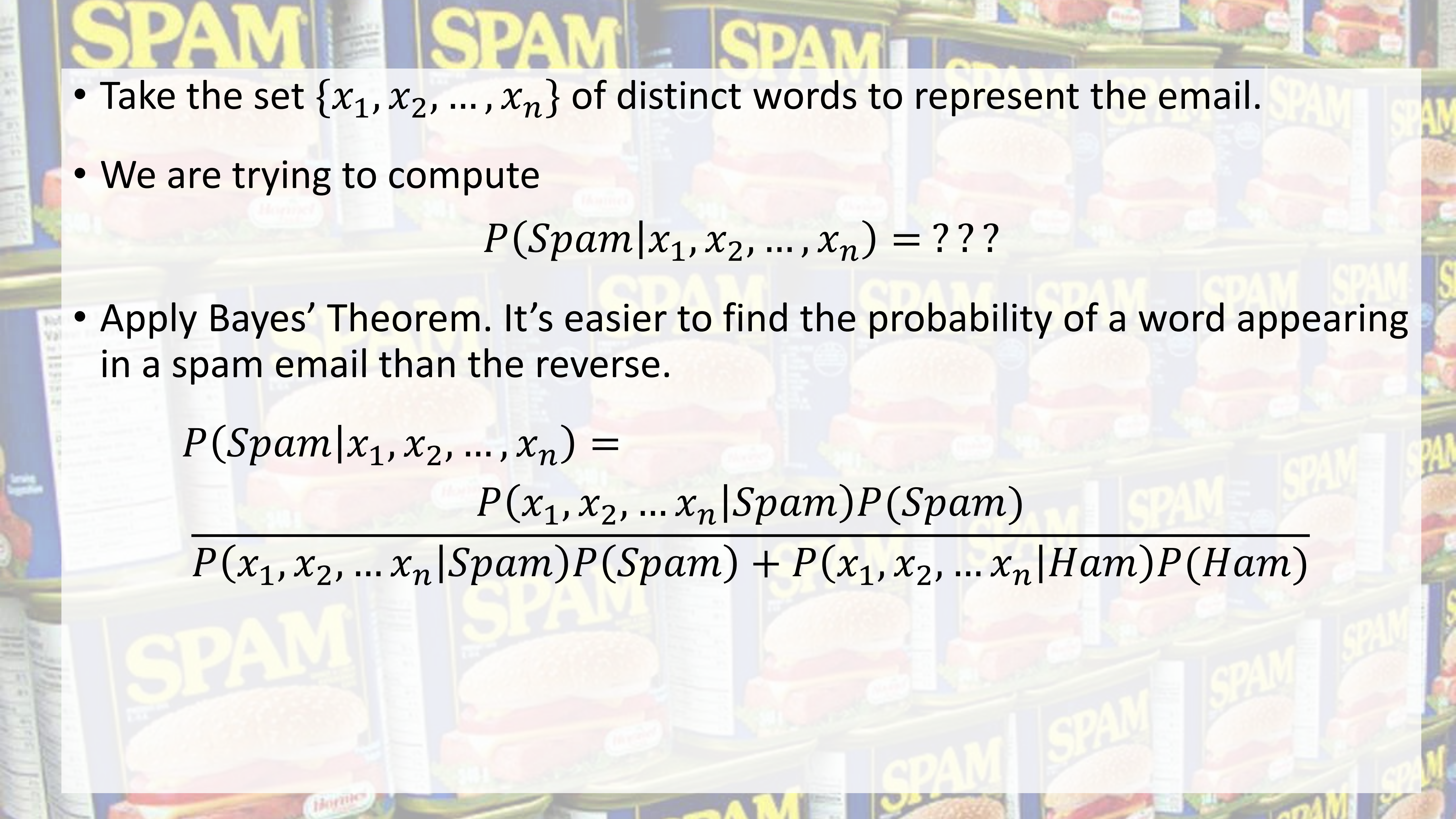
Dear Sir.

First, I must solicit your confidence in this transaction, this is by virture of its nature as being utterly confidencial and top secret…

$\rightarrow$

{top, secret, business, venture, dear, sir, first, I, must, solicit, your, confidence, in, this, transaction, is, by, virture, of, its, nature, as, being, utterly, confidencial, and}

Notice that there are no duplicate words

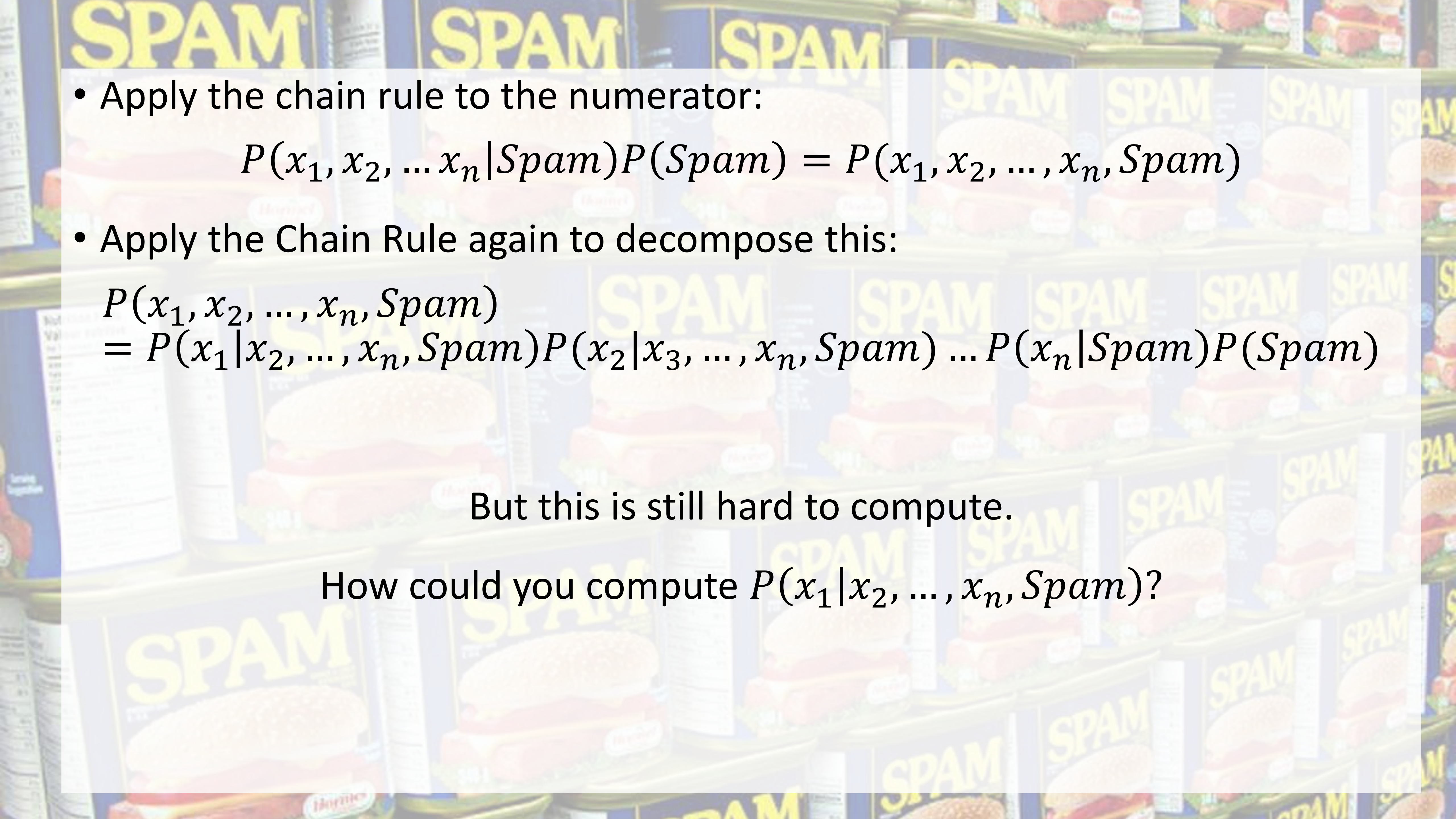


- Take the set $\{x_1, x_2, \dots, x_n\}$ of distinct words to represent the email.
- We are trying to compute

$$P(Spam|x_1, x_2, \dots, x_n) = ???$$

- Apply Bayes' Theorem. It's easier to find the probability of a word appearing in a spam email than the reverse.

$$P(Spam|x_1, x_2, \dots, x_n) = \frac{P(x_1, x_2, \dots x_n|Spam)P(Spam)}{P(x_1, x_2, \dots x_n|Spam)P(Spam) + P(x_1, x_2, \dots x_n|Ham)P(Ham)}$$



- Apply the chain rule to the numerator:

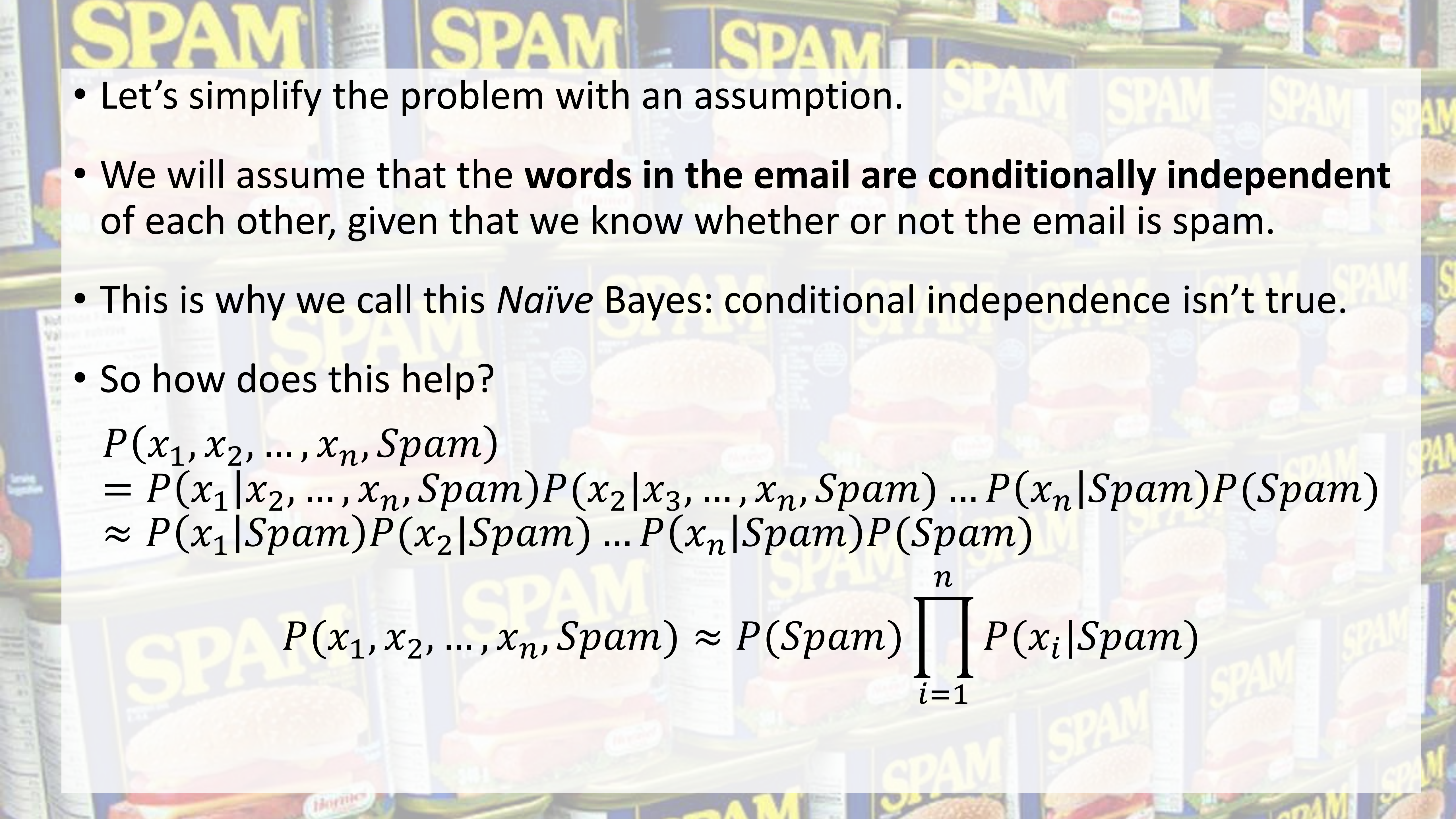
$$P(x_1, x_2, \dots x_n | Spam)P(Spam) = P(x_1, x_2, \dots, x_n, Spam)$$

- Apply the Chain Rule again to decompose this:

$$P(x_1, x_2, \dots, x_n, Spam)$$
$$= P(x_1 | x_2, \dots, x_n, Spam)P(x_2 | x_3, \dots, x_n, Spam) \dots P(x_n | Spam)P(Spam)$$

But this is still hard to compute.

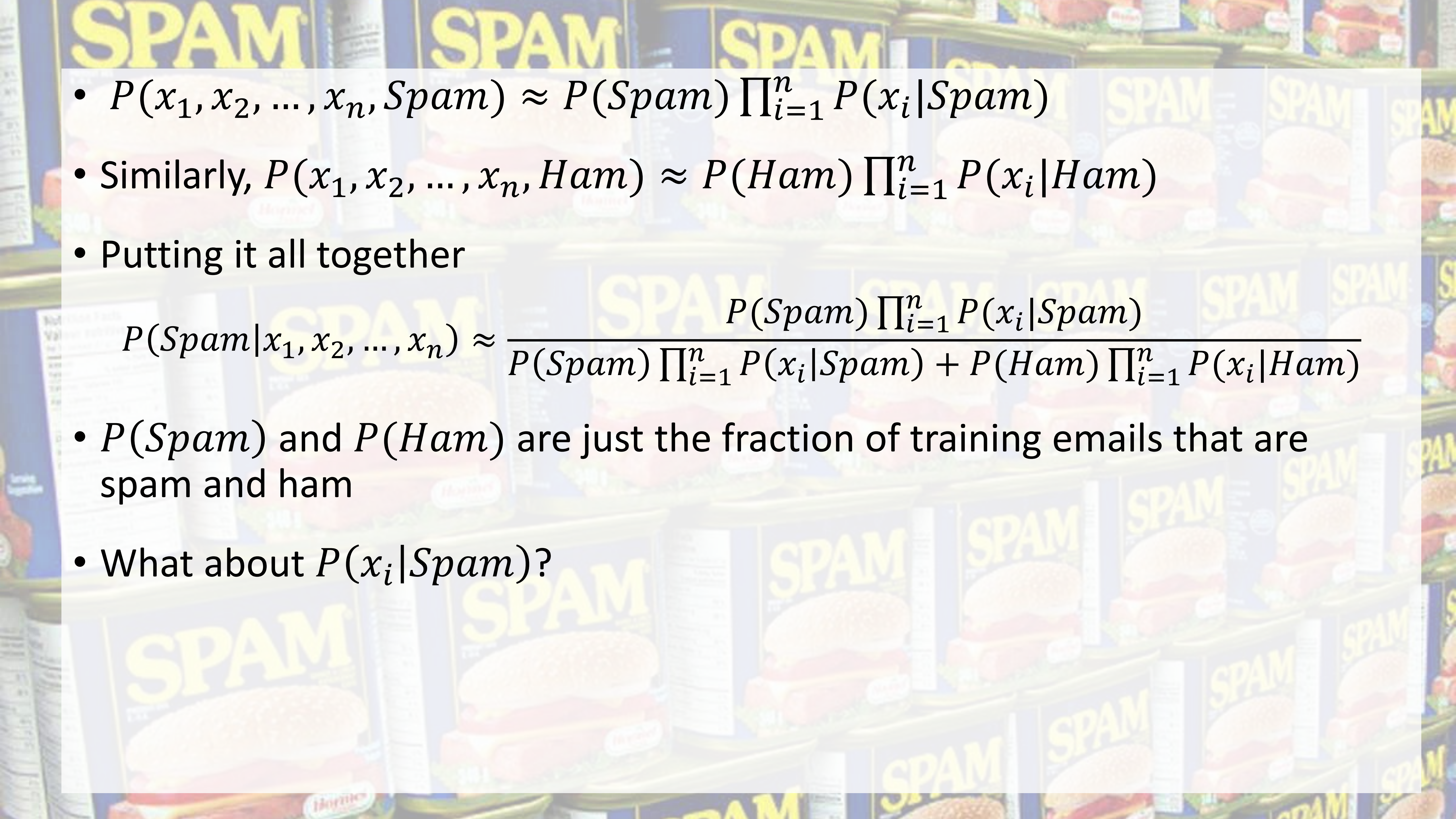
How could you compute $P(x_1 | x_2, \dots, x_n, Spam)$?



- Let’s simplify the problem with an assumption.
- We will assume that the **words in the email are conditionally independent** of each other, given that we know whether or not the email is spam.
- This is why we call this *Naïve* Bayes: conditional independence isn’t true.
- So how does this help?

$$
\begin{aligned}
&P(x_1, x_2, \dots, x_n, Spam) \\
&= P(x_1 | x_2, \dots, x_n, Spam) P(x_2 | x_3, \dots, x_n, Spam) \dots P(x_n | Spam) P(Spam) \\
&\approx P(x_1 | Spam) P(x_2 | Spam) \dots P(x_n | Spam) P(Spam)
\end{aligned}
$$

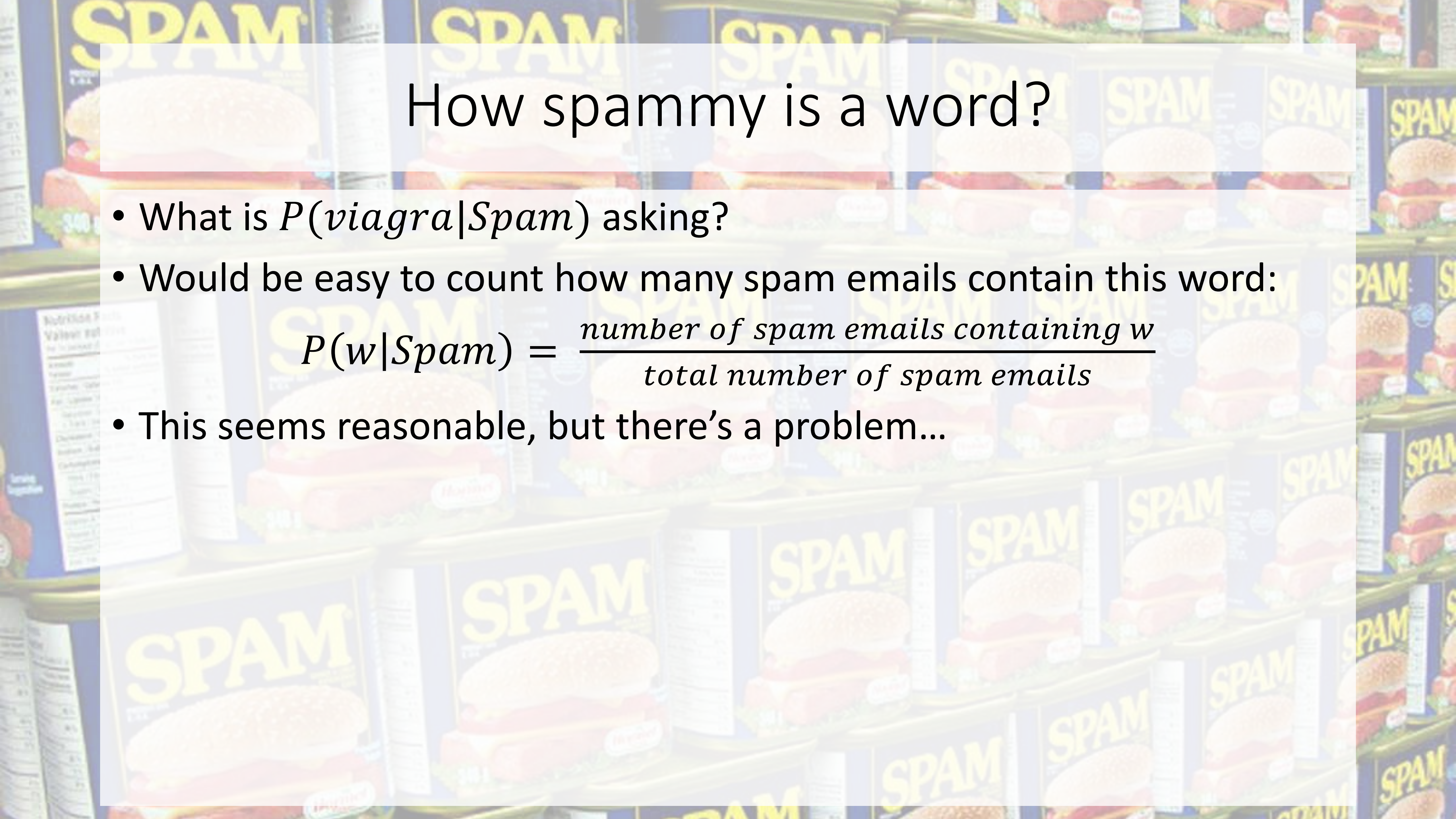
$$P(x_1, x_2, \dots, x_n, Spam) \approx P(Spam) \prod_{i=1}^{n} P(x_i | Spam)$$



- $P(x_1, x_2, \dots, x_n, Spam) \approx P(Spam) \prod_{i=1}^{n} P(x_i|Spam)$
- Similarly, $P(x_1, x_2, \dots, x_n, Ham) \approx P(Ham) \prod_{i=1}^{n} P(x_i|Ham)$
- Putting it all together

$$P(Spam|x_1, x_2, \dots, x_n) \approx \frac{P(Spam) \prod_{i=1}^{n} P(x_i|Spam)}{P(Spam) \prod_{i=1}^{n} P(x_i|Spam) + P(Ham) \prod_{i=1}^{n} P(x_i|Ham)}$$

- $P(Spam)$ and $P(Ham)$ are just the fraction of training emails that are spam and ham
- What about $P(x_i|Spam)$?



How spammy is a word?

- What is $P(viagra|Spam)$ asking?
- Would be easy to count how many spam emails contain this word:

$$P(w|Spam) = \frac{number\ of\ spam\ emails\ containing\ w}{total\ number\ of\ spam\ emails}$$

- This seems reasonable, but there’s a problem…

- Suppose the word *Pokemon* only appears in ham in the training data, never in spam

$$P(Pokemon|Spam) = 0$$

- Since the overall spam probability is the product of such individual probabilities, if any of those is 0, the whole product is 0
- Any email with the word *Pokemon* would be assigned a spam probability of 0

SUBJECT: Get out of debt!

Cheap prescription pills! Earn fast cash using this one weird trick! Meet singles near you and get preapproved for a low interest credit card! Pokemon

$\rightarrow$

definitely not spam, right?

- What can we do?

Laplace smoothing

- Crazy idea: what if we pretend we've seen every outcome once already?
- Pretend we've seen one more spam email *with* w, one more *without* w

$$P(w|Spam) = \frac{|spam\ emails\ containing\ w| + 1}{|spam\ emails| + 2}$$

- Then, $P(Pokemon|Spam) > 0$
- No one word will bias the overall probability too much
- General technique to avoid assuming that unseen events will never happen

Naïve Bayes Overview

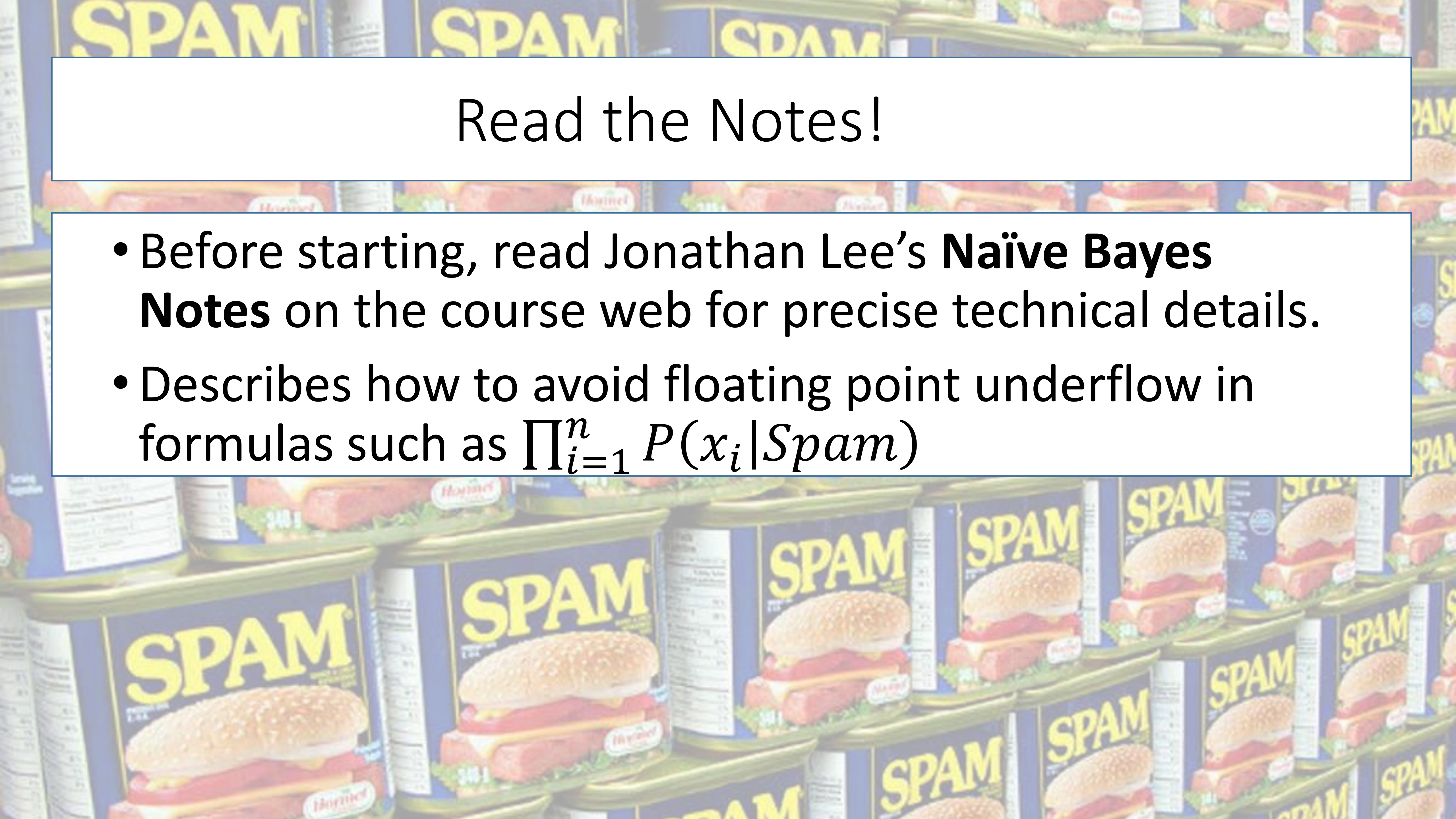
- For each word w in the spam training set, count how many spam emails contain w:

$$P(w|Spam) = \frac{|spam\ emails\ containing\ w| + 1}{|spam\ emails| + 2}$$

- Compute $P(w|Ham)$ analogously
- $P(Spam) = \frac{|spam\ emails|}{|spam\ emails| + |ham\ emails|}$, $P(Ham) = 1 - P(Spam)$
- For each test email with words $\{x_1, x_2, \dots, x_n\}$,

$$P(Spam|x_1, x_2, \dots, x_n) \approx \frac{P(Spam)\prod_{i=1}^{n} P(x_i|Spam)}{P(Spam)\prod_{i=1}^{n} P(x_i|Spam) + P(Ham)\prod_{i=1}^{n} P(x_i|Ham)}$$

Output “spam” iff $P(Spam|x_1, x_2, \dots, x_n) > 1/2$



Read the Notes!

- Before starting, read Jonathan Lee’s **Naïve Bayes Notes** on the course web for precise technical details.
- Describes how to avoid floating point underflow in formulas such as $\prod_{i=1}^{n} P(x_i|Spam)$