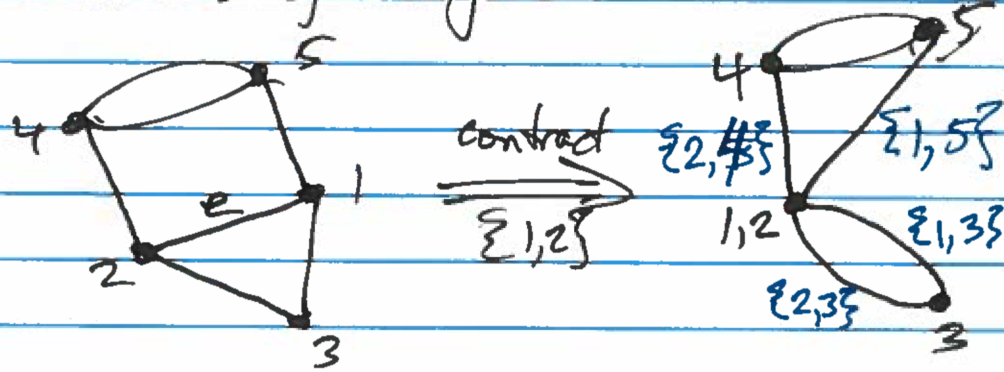
Karger's probabilistic algorithm:

1. Pick an edge $\{u,v\}$ randomly and uniformly and contract it by merging u and v. Retain multiple edges but remove self-loops.



2. Repeat step 1 until only 2 vertices remain. Output the set of edges between these 2 vertices as a cut of the original graph.

Every cut after a contraction is a cut of the original graph.
Let C be a min-cut of the original graph with k edges. Compute the probability that no edge in C is ever contracted, so that C is output.
A graph with n vertices and min-cut k must have at least $kn/2$ edges: every vertex has at least k edges incident on it (or else the min-cut would be $< k$), so there are at least kn "ends of edges" ~~at~~ and at least $kn/2$ edges.

Let E_i be the event that the i^{th} ~~contracted~~ edge is not in C, for $1 \le i \le n-2$, where n is the number of vertices in the original graph.

$P(\overline{E_1}) \le \frac{k}{kn/2} = \frac{2}{n}$, so $P(E_1) \ge 1 - \frac{2}{n}$

$P(\overline{E_2} | E_1) \le \frac{k}{k(n-1)/2} = \frac{2}{n-1}$, so $P(E_2|E_1) \ge 1 - \frac{2}{n-1}$

$P(\overline{E_i} | E_1 \cap E_2 \cap \cdots \cap E_{i-1}) \le \frac{k}{k(n-i+1)/2} = \frac{2}{n-i+1}$, so

$$P(E_i | E_1 \cap E_2 \cap \cdots \cap E_{i-1}) \ge 1 - \frac{2}{n-i+1}$$

$$P\left(\bigcap_{i=1}^{n-2} E_i\right) = P(E_1) P(E_2|E_1) \cdots P(E_{n-2} | E_1 \cap E_2 \cap \cdots \cap E_{n-3})$$

$$\ge \prod_{i=1}^{n-2}\left(1 - \frac{2}{n-i+1}\right) = \prod_{i=1}^{n-2} \frac{n-i-1}{n-i+1}$$

$$= \frac{\cancel{n-2}}{n} \cdot \frac{\cancel{n-3}}{n-1} \cdot \frac{\cancel{n-4}}{\cancel{n-2}} \cdots \frac{2}{\cancel{4}} \cdot \frac{1}{\cancel{3}} = \frac{2}{n(n-1)}$$

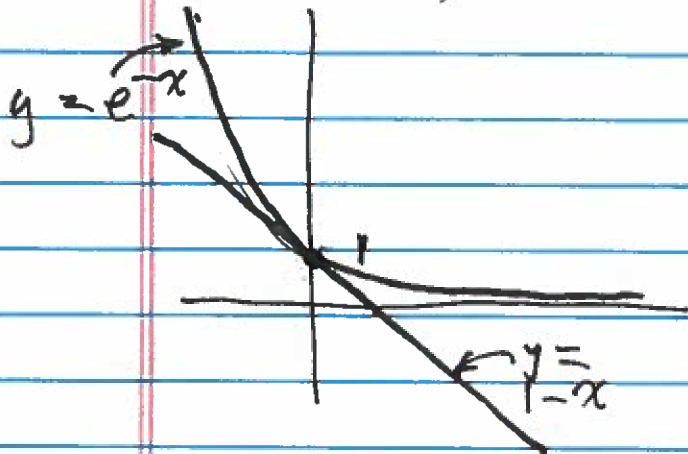
$$> \frac{2}{n^2}$$

This is the probability of success.
Repeat the algorithm ~~t~~ t times independently, and report the minimum cut of these t trials.
If we pick $t = sn^2/2$, the probability that C is not output in any of the t trials is

$$\le \left(1 - \frac{2}{n^2}\right)^t = \left(1 - \frac{2}{n^2}\right)^{sn^2/2} \le \left(\frac{1}{e}\right)^s$$

when $s = 14$, this is $< 10^{-6}$

For all x, $1-x \le e^{-x}$.



So $(1-x)^{1/x} \le e^{-1}$, so $(1-x)^{s/x} \le \left(\frac{1}{e}\right)^{s}$