Ex: Each day, your computer crashes with probability 10%, independent of every other day. What is the probability of ≥ 87 crash-free days in the next 100 days?

Let X be the number of crash-free days in the next 100 days.

$X \sim Bin(100, 0.9)$. $E[X] = 100 \times 0.9 = 90$, $Var(X) = 90 \times 0.1 = 9$

$$P(X \geq 87) = P(87 \leq X \leq 100) = P(86.5 < X < 100.5)$$

$$= P\left(\frac{86.5-90}{3} < \frac{X-90}{3} < \frac{100.5-90}{3}\right)$$

$$\cancel{\approx} P\left(-1.17 < \frac{X-90}{3} < 3.5\right)$$

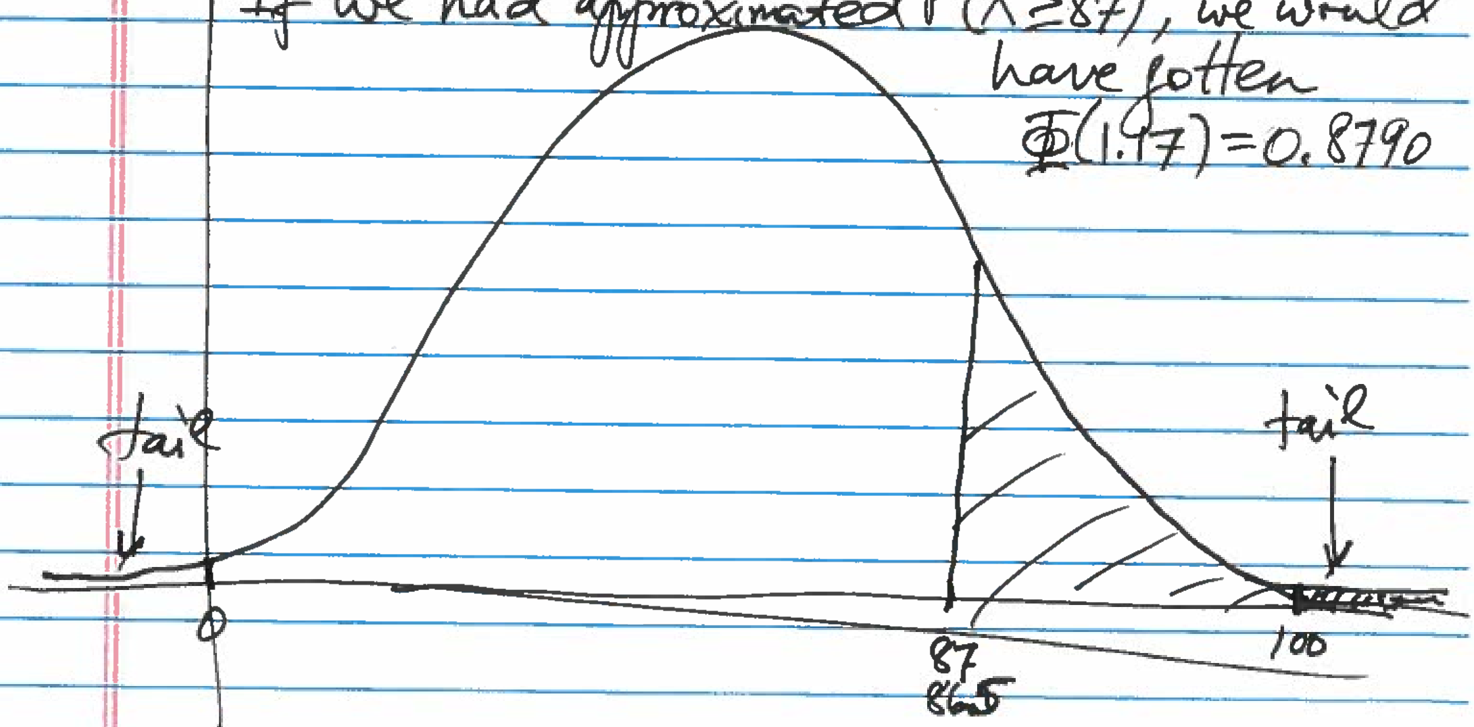
$$\approx \Phi(3.5) - \Phi(-1.17) \qquad \text{(CLT)}$$

$$= \Phi(3.5) - (1 - \Phi(1.17))$$

$$= \Phi(3.5) + \Phi(1.17) - 1$$

$$\cancel{\approx} 0.9998 + 0.8790 - 1 = 0.8788$$

If we had approximated $P(X \geq 87)$, we would have gotten $\Phi(1.17) = 0.8790$



Tail Bounds

Bound the probability of being far from the mean.

Markov's Inequality

Theorem: If X is a nonnegative random variable, then for any $\alpha > 0$,

$$P(X \geq \alpha) \leq \frac{E[X]}{\alpha}.$$

Equivalently, $P(X \geq kE[X]) \leq \frac{1}{k}$.

Ex: Suppose your expected ~~busin~~ business expense per day is \$1500. What is the probability that a given day's expense will be $\geq$ \$6000?

Let X be daily business expense.

$$P(X \geq 6000) \leq \frac{E[X]}{6000} = \frac{1500}{6000} = \frac{1}{4}$$

Proof: $E[X] = \sum_x x\, p_X(x) = \sum_{x<\alpha} x\, p_X(x) + \sum_{x \geq \alpha} x\, p_X(x)$

$$\geq 0 + \sum_{x \geq \alpha} \alpha\, p_X(x) = \alpha \sum_{x \geq \alpha} p_X(x)$$

$$= \alpha P(X \geq \alpha)$$

so $P(X \geq \alpha) \leq \frac{E[X]}{\alpha}$