Exponential (continuous) r.v.
Time until the next event, where events happen independently at rate λ per unit time.
(Poisson measures number of events in a given unit of time.)

Ex: Radioactive decay: time to next emission of an α particle.
Servers: time until the next packet arrives at a server.

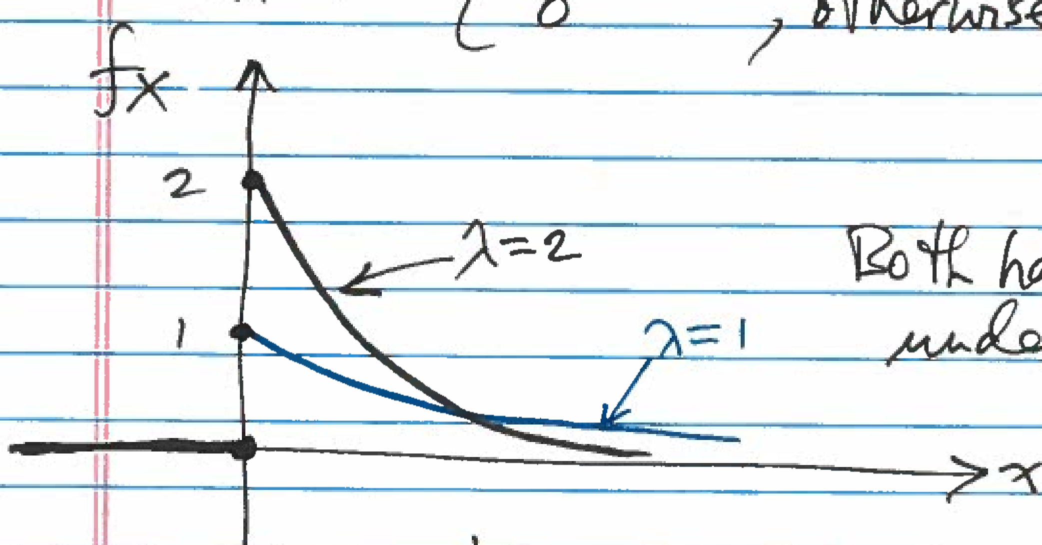
$X \sim Exp(\lambda)$

$P(X > t) = e^{-\lambda t}$

For $t \geq 0$, $F_X(t) = P(X \leq t) = 1 - P(X > t) = 1 - e^{-\lambda t}$

$f_X(x) = \frac{d}{dx} F_X(x) = \frac{d}{dx}(1 - e^{-\lambda x}) = +e^{-\lambda x} \cdot \lambda$

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$



Both have area 1 under the curve.

$E[X] = \frac{1}{\lambda}$, using integration by parts

$Var(X) = \frac{1}{\lambda^2}$

Memorylessness of $X \sim Exp(\lambda)$:

For $s, t > 0$, $P(X > s+t \mid X > s) = P(X > t)$

Exponential is continuous analog of the geometric: both measure time until the next event. The geometric measures time discretely, the exponential measures time continuously.

$X \sim geo(p)$ also has memorylessness property.

Relationship of ~~memorle~~ memorylessness to independence:

Let E = no emissions up to time s.

Let F = " " in $[s, s+t]$.

Then E and F are independent.