$$E[X] = \sum_x x\, p_X(x)$$

$$E[g(X)] = \sum_x g(x) p_X(x), \text{ for any function } g$$

Theorem: For any constants a and b,

$$E[aX+b] = a E[X] + b.$$

Proof:

$$E[aX+b] = \sum_x (ax+b) p_X(x)$$

$$= \sum_x \left(a x p_X(x) + b\, p_X(x)\right)$$

$$= a \sum_x x p_X(x) + b \sum_x p_X(x)$$

$$= a E[X] + b$$

Ex: A casino charges \$1 to play the following game. They flip a coin that has prob 1/8 of ~~height~~ heads independently, and pay you 12¢ for each flip up to and including first head. Do you expect to win or lose money?

Let X be # of flips up to and including first head. Your expected gain is

$$E[12X - 100] = 12 E[X] - 100 = 12 \cdot \frac{1}{1/8} - 100$$

$$= -4$$