In the case of equally likely outcomes,

$$P(E) = \frac{|E|}{|\Omega|}. \quad \text{(Axiom 3)}.$$

$$P(E) = P\Big(\bigcup_{a\in E} a\Big) = \sum_{a\in E} P(a) = \sum_{a\in E} \frac{1}{|\Omega|} = \frac{|E|}{|\Omega|}$$

Ex. Assume ♣J is the face-up trump. What is the probability that your initial hand has 0 trumps?

E = set of hands with 0 trumps

Ω = set of all possible hands

$$|E| = \binom{15}{5}, \quad |\Omega| = \binom{19}{5}$$

$$P(E) = \frac{|E|}{|\Omega|} = \frac{\binom{15}{5}}{\binom{19}{5}} = \frac{15\cdot 14\cdot 13\cdot 12\cdot 11/5!}{19\cdot 18\cdot 17\cdot 16\cdot 15/5!} \approx 0.258$$

Ex: Suppose your 5-card hand is dealt before a trump is turned up. What is the probability that you have at least one marriage?

E = set of hands with ≥ 1 marriage.

Ω = set of all possible hands. $|\Omega| = \binom{20}{5}$

$$P(E) = \frac{\binom{4}{1}\binom{18}{3} - \binom{4}{2}\binom{16}{1}}{\binom{20}{5}} \approx 0.204$$

Ex: Assume 365 birthdays are equally likely. What is the probability that, out of n people, none share the same birthday?

$$|\Omega| = 365^n$$

E = assignment of distinct birthdays to all n

$$|E| = P(365, n) = 365\cdot 364 \cdots (365-n+1)$$

$$P(E) = \frac{365 \cdot 364 \cdots (365-n+1)}{365^n} = p_n$$

$n = 23$: $p_{23} < 0.5$

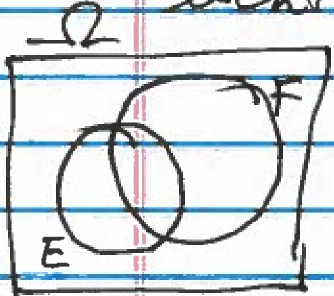
$n = 77$: $p_{77} < 1/5000$

$n = 100$: $p_{100} < 1/3\times 10^{15}$

Ex: n chips are manufactured, d of them defective. k chips are selected randomly for testing. Let E be that at least 1 of the k chips is defective. What is $P(E)$? Ω = all possible sets of k chips.

$$P(\overline{E}) = \frac{\binom{n-d}{k}}{\binom{n}{k}}, \quad P(E) = 1 - P(\overline{E}) = 1 - \frac{\binom{n-d}{k}}{\binom{n}{k}}$$

Conditional probability of E given F, where E and F are events, is written $P(E|F)$, where $F \neq \emptyset$. $P(E|F)$ is the probability of event E, given that F occurred.



Sample space is reduced to F, and the event is reduced to $E \cap F$.

With equally likely outcomes,

$$P(E|F) = \boxed{\frac{|E\cap F|}{|F|}}$$

$$= \frac{|E\cap F|/|\Omega|}{|F|/|\Omega|} = \boxed{\frac{P(E\cap F)}{P(F)}}$$

true even if outcomes not equally likely

<u>Ex:</u> Roll a fair die. What is $P(5 \mid odd)$

$E = \{5\}$, $F = \{1,3,5\}$

$$P(E|F) = \frac{|E \cap F|}{|F|} = \frac{|E|}{|F|} = \frac{1}{3}$$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/6}{1/2} = \frac{1}{3}$$