1. How many subsets does an n-element set have?

2. How many 4-character passwords, each character either a lowercase letter or digit?

$$36^4 \approx 1.7 \times 10^6$$

3. How many if no character is repeated?

$$P(36,4) = 36 \cdot 35 \cdot 34 \cdot 33 \approx 1.4 \times 10^6$$

Permutations: order matters

How many sequences are there of length n, where each character is from $\{1, 2, \ldots, n\}$ and each character appears exactly once?

$$P(n,n) = n! = n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1$$

$$0! = 1$$

k-permutations: How many sequence of length $k \leq n$, where each character is from $\{1, 2, \ldots, n\}$ and each character appears at most once?

$$P(n,k) = n \cdot (n-1) \cdot (n-2) \cdots (n-k+1) = \frac{n!}{(n-k)!}$$

What if the n characters are not all distinct?

Ex: How many permutations are there of the letters in DOGGY? $\frac{5!}{2!}$

$OG_1DYG_2 = OG_2DYG_1$

GODOGGY $\frac{7!}{2!3!1!1!}$

Combinations : order doesn't matter.

Ex- Your elf-lord avatar can pick up any 3 of the following weapons:
sword, axe, knife, ring, laptop
How many combinations of exactly 3?

$$\frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = 10$$