CSE 312: Foundations of Computing II
Additional Exercises #1: Sets, Functions, Permutations

Note: These exercises are strictly for your own benefit, in case you need extra practice for exams and homework.

Several exercises below deal with a "standard" 52-card deck, such as is used in the games of bridge and poker. This deck consists of 52 cards divided into 4 suits of 13 cards each. The 4 suits are (black) spades ♠, (red) hearts ♡, (black) clubs ♣, and (red) diamonds ◇. The 13 cards ("ranks") of each suit are 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A.

1. A chef is preparing desserts for the week, starting on a Sunday. On each day, only one of five desserts (apple pie, cherry pie, strawberry pie, pineapple pie, and cake) may be served. On Thursday there is a birthday, so cake must be served that day. On no two consecutive days can the chef serve the same dessert. How many dessert combinations are there for the week?

2. A store has 4 books, 14 movies, 6 toys, and 5 posters. In how many ways can a customer buy exactly 1 item from each of exactly 3 categories?

3. In how many different ways can you arrange seven people around a circular table?

4. Suppose that 8 people, including you and a friend, line up for a picture. In how many ways can the photographer organize the line if she wants to have fewer than 2 people between you and your friend?

5. Your CSE 312 teaching staff (Professor Rao and 6 TAs) lines up for a picture. How many possible arrangements are there with Professor Rao not at either end of the line?

6. How many ways are there to permute the 8 letters A, B, C, D, E, F, G, H so that A is not at the beginning and H is not at the end?

7. There are 40 seats and 40 students in a classroom. Suppose that the front row contains 10 seats, and there are 5 students who must sit in the front row in order to see the board clearly. How many seating arrangements are possible with this restriction?

8. Permutations of objects, some of which are indistinguishable.

   (a) How many permutations are there of the letters in DAWGY?

   (b) How many permutations are there of the letters in DOGGY?

   (c) How many permutations are there of the letters in GODOGGY?

9. A bridge hand consists of 13 cards dealt from a shuffled standard deck of 52 cards. Given a bridge hand consisting of 5 spades, 2 hearts, 3 diamonds, and 3 clubs, in how many ways can the hand be arranged so that the cards of each suit are together ...

    (a) ... but not necessarily sorted by rank within each suit?

    (b) ... and each suit is sorted in ascending rank order?

    (c) ... and each suit is sorted in ascending rank order and the suits are arranged so that the suit colors alternate?

10. Suppose two cards are drawn in order from a bridge deck. In how many ways can the first card be a diamond and the second card a jack?

11. Rabbits Peter and Pauline have three offspring: Flopsie, Mopsie, and Cotton-tail. These five rabbits are to be distributed to four different pet stores so that no store gets both a parent and a child. It is not required that every store gets a rabbit. In how many different ways can this be done?

12. You have a triangular prism with top and bottom both being congruent equilateral triangles and the three sides being congruent rectangles. If you pick 5 out of 7 different colors, one to paint each of the 5 faces, how many differently painted triangular prisms can you get? Just rotating the prism does not constitute a different color scheme.

13. How many ways are there to seat 10 people, consisting of 5 couples, in a row of 10 seats if ...

    (a) ... the seats are assigned arbitrarily?

    (b) ... all couples are to get adjacent seats?

    (c) ... the seats are assigned arbitrarily, except that one couple insists on not sitting in adjacent seats?