CSE 312: Foundations of Computing II
Quiz Section #1: Sets, Functions, Counting

Note: The exercises marked as **(not covered yet)** use combinations, which have not been discussed in lecture yet as of Friday, Jan 5. They will not appear on Homework 1.

Note: If you would like more practice for exams and homework, there is another file on the website containing additional problems.

Review: Main Theorems and Concepts

1. **Sets:** A **set** is a collection of distinct objects, called elements. Some important sets include:

   (a) The integers $\mathbb{Z} = \{..., -2, -1, 0, 1, 2, ...\}$

   (b) The naturals $\mathbb{N} = \{1, 2, 3, ...\}$

   (c) The rationals $\mathbb{Q} = \{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\}$

   (d) The reals $\mathbb{R}$

   (e) The first n naturals $[n] = \{1, 2, ..., n\}$

2. **Sequences:** A **sequence** is an *ordered* list of elements of a set (can have duplicates). For example, $1, -3, 2, -4, 2, -3$ is a sequence.

3. **Functions:** Let A, B be sets, called the **domain**, and **codomain**, respectively. A **function (or map)** $f : A \to B$ assigns each $a \in A$ to a unique value $b \in B$. We denote the value f maps a to as $f(a) \in B$.

   (a) The **range** of f is denoted $f(A) = \{f(a) : a \in A\} \subseteq B$

   (b) A function is **injective (or one-to-one)** if $\forall x, y \in A$ with $x \neq y$, it must be that $f(x) \neq f(y)$. Equivalently, if $f(x) = f(y)$, then $x = y$.

   (c) A function is **surjective (or onto)** if $f(A) = B$. That is, $\forall b \in B, \exists a \in A, f(a) = b$

   (d) A function is **bijective** if it is both injective and surjective. That is, for all $b \in B$, there is a *unique* $a \in A$ with $f(a) = b$.

   (e) Let $X \subseteq \mathcal{U}$ be a subset of some universal set. The **indicator function** for X is the function $1_X : \mathcal{U} \to \{0, 1\}$ such that $1_X(x) = 1$ if $x \in X$ and $1_X(x) = 0$ if $x \notin X$

4. **Product Rule:** Suppose there are m_1 possible outcomes for event A_1, then m_2 possible outcomes for event $A_2, \ldots,$ m_n possible outcomes for event A_n. Then there are ________________________ possible outcomes overall.

5. **Number of ways to order n distinct objects:** ________________________

6. **Permutations** (number of ways to linearly arrange k objects out of n distinct objects, when the order of the k objects matters):

   ____

Exercises

Several exercises below deal with a “standard” 52-card deck, such as is used in the games of bridge and poker. This deck consists of 52 cards divided into 4 suits of 13 cards each. The 4 suits are (black) spades ♠, (red) hearts ♡, (black) clubs ♣, and (red) diamonds ♢. The 13 cards (“ranks”) of each suit are 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A.

1. **(Not covered yet)** How many ways are there to select 5 cards from a standard deck of 52 cards, where the 5 cards contain cards from at most two suits, if:

   (a) order does not matter

   (b) order matters

2. **(Not covered yet)** Consider a set of 25 people that form a social network. (The structure of the social network is determined by which pairs of people in the group are friends.) How many possibilities are there for the structure of this social network?

3. Suppose we have 3 diamonds and 3 hearts from a standard deck. How many ways are there to arrange the cards if they have to alternate suit?

4. How many ways are there to choose three initials that have two being the same or all three being the same?

5. A license plate has the form AXYZBCD, where A, B, C, and D are digits and X, Y, and Z are upper case letters. What is the number of different license plates that can be created?