NAME: ____________________________

CSE 312
Computational Complexity Theory
Practice Midterm 1, Winter 2018

31 January 2018

DIRECTIONS:

- Answer the problems on the exam paper.
- You are allowed to use a cheat sheet and a calculator.
- If you need extra space use the back of a page
- You have 50 minutes to complete the exam.
- Please do not turn the page until you are instructed to do so.
- Good Luck!

1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
Total	/60

1. (10 points, 5 each) A small scale distributed file system consists of 12 servers. We number them from 1 to 12 respectively. According to historical data, each server crashes independently with probability 1/100 every hour. Answer the following for one particular hour:

   - What is the probability both the 3rd server and the 6th server crash?
   - The entire system will continue to function as long as more than 3 of the servers are working. What is the probability that the entire system will fail?

2. (10 points) Theres a popular social network where two people are considered to be connected if both of them add each other as *friends*. The structure of this social network is determined by which pairs of people are *friends*. If there are n active users on the social network, how many possibilities are there for the structure of this social network?

3. (10 points) I have a pile of 6 identical-looking coins:

   - 3 of them are fair coins with $p(heads) = 1/2$.
   - 2 of them are biased such that $p(heads) = 1/3$.
   - 1 of them is biased such that $p(heads) = 2/3$.

   Suppose I draw a coin uniformly at random from the six coins. Then I flip the coin 6 times, and it comes up heads 4 times. What is the probability that the coin I flipped was one of the fair coins? No need to simplify your answer.

4. (10 points) There is a train with 3 carriages that are initially empty. It arrives at a train station where 20 people are waiting to get on. Each person chooses one of the carriages. How many ways are there for people to get on the train so that none of the carriages are empty? You may leave your answer as a simplified expression.

5. (10 points) Suppose we throw n balls into n bins with the probability of a ball landing in each of the n bins being equal. What is the expected number of empty bins?

6. (15 points, 5 each) Suppose we repeatedly toss a fair coin until we see r heads, and then stop. Let X be the number of coin tosses in total.

   (a) What is $\mathbb{E}[X]$?

   (b) For any number t, what is $p(X = t)$ as a function of t, r?

   (c) What is $\text{Var}[X]$?