Bayes' Classifier

\[ H = S^c \]

\( S \): event that email is spam

\( V \): event that email contains "viagra"

\[
p(S|V) = \frac{p(V|S) \cdot p(S)}{p(V)}
\]

\[
= \frac{p(V|S) \cdot p(S)}{p(V|S) \cdot p(S) + p(V|H) \cdot p(H)}
\]

\[
p(M|E) = \frac{p(E|M) \cdot p(M)}{p(E)}
\]

Words in email

\( x_1, x_2, \ldots, x_n \)

\[
p(s|x_1, \ldots, x_n) = \frac{p(x_1, \ldots, x_n|S) \cdot p(S)}{p(x_1, \ldots, x_n|S) \cdot p(S) + p(x_1, \ldots, x_n|H) \cdot p(H)}
\]

Assumption

Words are independent!

\[
p(x_1, \ldots, x_n|S) \cdot p(S)
\]

\[
= p(S) \cdot p(x_1|S) \cdot p(x_2|S) \cdots p(x_n|S)
\]

\[
p(x_1, \ldots, x_n|S) \cdot p(S) + p(x_1, \ldots, x_n|H) \cdot p(H)
\]
Smoothing

Goal: $\forall x \in \mathcal{X}$ for all words $x$

$p(x|S) > 0$, $p(x|H)$

To estimate

$p(w|S)$: $\frac{\text{# spam emails containing } w}{\text{# spam emails}}$

$\Rightarrow$ smooth

$\Rightarrow \frac{\text{# spam emails with } w + 1}{\text{# spam emails} + 2}$

Words in email

$x_1, x_2, \ldots, x_n$

$p(s|x_1, \ldots, x_n) = \frac{p(x_1, \ldots, x_n|S) \cdot p(S)}{p(x_1, \ldots, x_n|S) \cdot p(S) + p(x_1, \ldots, x_n|H) \cdot p(H)}$

Assumption

(words are independent!)

$p(x_1, \ldots, x_n|S) \cdot p(S)$

$= p(S) \cdot p(x_1|S) \cdot p(x_2|S) \cdots p(x_n|S)$

$\Rightarrow$

$p(S) \cdot p(x_1|S) \cdots p(x_n|S)$

$p(S) \cdot p(x_1|S) \cdots p(x_n|S) + p(x_1|H) \cdots p(x_n|H) \cdot p(H)$
Monty Hall Game show

2 doors $\rightarrow$ goat
1 door $\rightarrow$ car

1. Contestant picks a door $\Box$

2. Host picks a random goat-door among two not picked by contestant

3. Contestant stay or switch.

$C_1, C_2, C_3$ : events that car is behind $\Box$, $\Box$, $\Box$

$H_2$ : host picks door $\Box$

$H_3 = H_2^c$ : host picks door $\Box$

$\overline{H_3}$ happened.

$p(C_1 | H_3)$ vs $p(C_2 | H_3)$

$p(C_2 | H_3) = \frac{p(H_3 | C_2) \cdot p(C_2)}{p(H_3)}$

$p(H_3 | C_2) = 1 \quad p(C_2) = \frac{1}{3}$

$p(H_3) = \frac{1}{2} \quad p(H_2) + p(H_3) = 1$

$p(H_3) = p(H_3 | C_1) \cdot p(C_1) + p(H_3 | C_i^c) \cdot p(C_i^c)$.
\( C_1, C_2, C_3 \): events that car is behind

\( H_2 \): host picks door 2

\( H_3 = H_2^c \): host pick door 3

\( H_3 \) happened.

\[ p(C_1 | H_3) \] vs \[ p(C_2 | H_3) \]

\[ p(C_2 | H_3) = \frac{p(H_3 | C_2) \cdot p(C_2)}{p(H_3)} \]

\[ p(H_3 | C_2) = 1 \quad p(C_2) = \frac{1}{3} \]

\[ p(H_3) = \frac{1}{2} \quad p(H_2) + p(H_3) = 1 \]

\[ p(H_3) = p(H_3 | C_1) \cdot p(C_1) + p(H_3 | C_1^c) \cdot p(C_1^c) \]

\[ p(C_2 | H_3) = \frac{1}{\frac{1}{2}} \cdot \left( \frac{2}{3} \right) = \frac{2}{3} \]

\[ p(C_1 | H_3) + p(C_2 | H_3) = 1 \quad \Rightarrow \quad p(C_1 | H_3) = \frac{1}{3} \]
Study 1:
Acceptance rates are same.

Study 2:
In every major, acceptance rate for women was lower than for men.

W: event applicant is woman
M: " " " man
A: " " " is admitted

\[ P(A|W) = P(A|M) \]

2 majors:
X: event applicant in Computer science
Y: " app in statistics

Study 2:
\[ P(A|W,X) < P(A|M,X) \]
\[ P(A|W,Y) < P(A|M,Y) \]