$$X = \underbrace{X_1 + X_2 + \cdots + X_n}_{\text{indep.}} \quad \in \{0,1\}$$

$$\mathbb{E}[X] = \mu$$

Thm: $\forall \delta > 0$,

$$p(X \geq (1+\delta)\mu) \leq e^{\frac{-\delta^2 \mu}{2+\delta}}$$

$$p(X \leq (1-\delta)\mu) \leq e^{\frac{-\delta^2 \mu}{2}}.$$

when $\delta < 1$

$$\leq e^{\frac{-\delta^2 \mu}{3}}$$

## Distributed Load Balancing

$k$ servers

$n$ jobs

Assign each job to a unif. random server

$p($max. load $\gg n/k)$ ?

$X_1$ : # jobs received by 1'st server

$X_2$ :

$\vdots$ :

$X_k$ : " " k'th server

$$p(\max_j X_j \gg n/k) \;?$$

$$X_1 = Y_1 + Y_2 + \cdots + Y_n$$

$$Y_i = \begin{cases} 1 & \text{if } i^{\text{th}} \text{ job goes to server } 1 \\ 0 & \text{o.w.} \end{cases}$$

$Y_i$ is Bernoulli with $p = 1/k$.

$$\mathbb{E}[Y_i] = 1/k$$

$$\mathbb{E}[X_1] = n/k$$

---

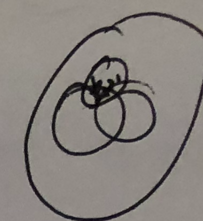
$$p\Big(X_1 \geq \overbrace{n/k + 3\sqrt{\frac{n \ln k}{k}}}^{M}\Big)$$

$$= p\Big(X_1 \geq \underbrace{\frac{n}{k}}_{\mu}\Big(1 + 3\underbrace{\sqrt{\frac{k \ln k}{n}}}_{\delta}\Big)\Big)$$

$$\leq e^{-\frac{\left(3\sqrt{\frac{k \ln k}{n}}\right)^2}{3} \cdot \frac{n}{k}}$$

$$= e^{-3 \ln k} = \left(e^{\ln k}\right)^{-3}$$

$$= \frac{1}{k^3}$$



$$p(X_1 \geq M \text{ OR } X_2 \geq M \text{ OR } \cdots X_k \geq M)$$

$$\leq p(X_1 \geq M) + p(X_2 \geq M) \cdots + \cdots + p(X_k \geq M)$$

$$\leq \frac{1}{k^3} + \frac{1}{k^3} + \cdots + \frac{1}{k^3} \leq \frac{1}{k^2}.$$

---

$k = 1000$, $n = 10^6$

$M = 1249.38$ $\quad p(\ast) \leq \frac{1}{10^9}$

## Chebyshev

$$p(|X-\mu| > \text{large}) = ?$$

$$\mathbb{E}[X^2]$$

$$p(X \geq \alpha)$$

$$= p(X^2 \geq \alpha^2) \leq \frac{\mathbb{E}[X^2]}{\alpha^2}$$

---

$$p(X \geq \alpha)$$

$$= p(e^X \geq e^\alpha) \leq \frac{\mathbb{E}[e^X]}{e^\alpha}$$

$$\mathbb{E}[e^X] = \mathbb{E}[e^{X_1+X_2+\cdots X_n}]$$

$$= \mathbb{E}[e^{X_1} e^{X_2} \cdots e^{X_n}]$$

$$= \mathbb{E}[e^{X_1}]\,\mathbb{E}[e^{X_2}]\cdot\mathbb{E}[e^{X_3}]\cdots$$

# Max Likelihood Est

1. Guess a model
2. ~~Used~~ Use data to compute params.

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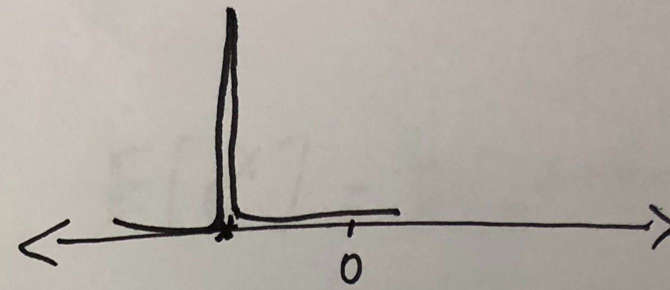
**Driver 1**

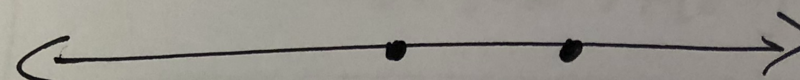
All 3 ★'s

**Driver 2**

50% 5 ★'s

50% 1 ★'s



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$$\frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}} \cdot e^{\frac{-(x-\mu)^2}{2}}$$

Suppose: $\sigma = 1 \quad \mu = ?$

Data: $x_1, x_2, \ldots, x_n$

$$L(x_1, x_2, \ldots, x_n \mid \theta) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} e^{-\frac{(x_i-\theta)^2}{2}}$$

Goal: Find $\theta$ to maximize likelihood.