

Pen 1

Pen 2

$$S, |S| = n$$

$$|S \times S| = n \times n = n^2$$

$$|S^k| = \underbrace{|S \times S \times \dots \times S|}_{k \text{ times}} = \underbrace{n \times n \times \dots \times n}_{k \text{ times}} = n^k$$

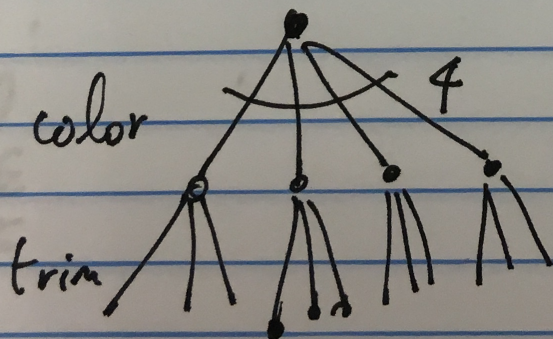
X ————— X

Car 4 choices for color

x  
3 choices for trim

= ~~15~~ 12 choices overall.

Product rule



How many subsets  $S \subseteq [n] = \{1, 2, \dots, n\}$ ?

$$\forall e \in \{0, 1\}^n$$

$$\# \text{ sets} = \underbrace{2 \times 2 \times \dots \times 2}_{n \text{ times}} = 2^n = |2^{[n]}|$$



S: 0 1 0 1 1 0 0 1 1 0



4 sections - can have multiple TAs > 0  
6 TAs

How many assignments s.t. every section gets  $\geq 1$  TA?

sections: [4]

- A: set of sections taught by Kaithya
- B: " " " Alex
- C: " " " Joshua
- D: " " "
- E: " " "
- F: " " "

Wrong

~~$2^4 \times 2^4 \times \dots \times 2^4 = (2^4)^6 = 2^{24}$~~

↓                      ↓  
opt for A      opt for B

set of TAs teaching 1<sup>st</sup> section

↓

A :	0	0	1	0	<del>0</del>
B :	1	0	1	0	
C :	0	1	0	1	
D :	1	1	1	1	
E :	0	0	0	0	
F :	1	1	0	1	

$S_1, S_2, S_3, S_4$  : set of TA's teaching each section

# choices for  $S_1 = 2^6 - 1$   
↳ not allowed  $\phi$ .  
# of subsets of  $[6]$

∴  
# choices for  $S_4 = 2^6 - 1$

Total # choices =  $(2^6 - 1) \cdot (2^6 - 1) \cdot (2^6 - 1) \cdot (2^6 - 1)$   
= 15,752,961

$C: \quad 0 \ 1 \ 0 \ 1$   
 $B: \quad 1 \ 0 \ 1 \ 0$   
 $A: \quad 0 \ 0 \ 1 \ 0$

5 books  
 distribute to  
 3 children

How many ways?

$[S] \supseteq \begin{array}{l} A: \text{ set of books of child 1} \\ B: \\ C: \end{array} \quad |$

WRONG  
 $2^5$   
 $\times 2^5 = 2^{15}$   
 $2^5$

$\downarrow \downarrow \downarrow \downarrow \downarrow$   
 $A: \quad 0 \ 1 \ 1 \ 0 \ 0$   
 $B: \quad 1 \ 1 \ 0 \ 0 \ 1$   
 $C: \quad 0 \ 0 \ 1 \ 1 \ 1$

$3 \times 3 \times 3 \times 3 \times 3 = 3^5$  ways.

How many rearrangements of

GRAPEFRUIT ?

$$10 \times 9 \times 8 \times \dots \times 1 = 10!$$

GRAPEFRUIT ?

$$10!$$

GRAPEFRUIT  
GRAPEFRUIT

GRAPEFRUIT

$$\frac{10!}{2!}$$

$$2!$$

CAAATTLE

$$\frac{8!}{3!}$$

$$3!$$

5  
PPFFLENN

$$\frac{9!}{2! \cdot 3! \cdot 2!}$$

X ————— X

$P(n, k)$  : # of ways to pick a sequence  
of length  $k$  with distinct elements  
from  $n$  things

$$= \frac{n!}{(n-k)!}$$

X ————— X  
# 5 letter words with distinct letters

$$26 \times 25 \times 24 \times 23 \times 22$$

$$\frac{26!}{2!} \quad ACDEL, \dots$$