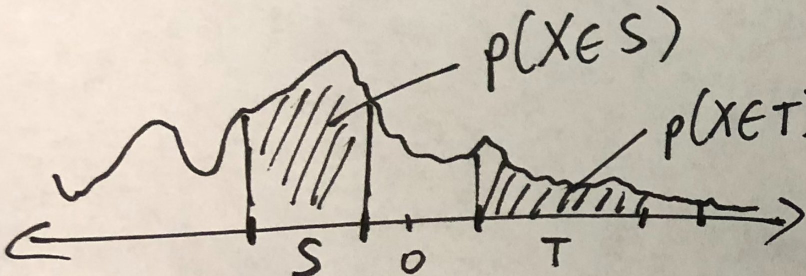
pdf: probability density function

$f(x)$

- For all x, $f(x) \geq 0$
- $\int_{-\infty}^{\infty} f(x)\, dx = 1$



Example:

Uniform between 0 and 1

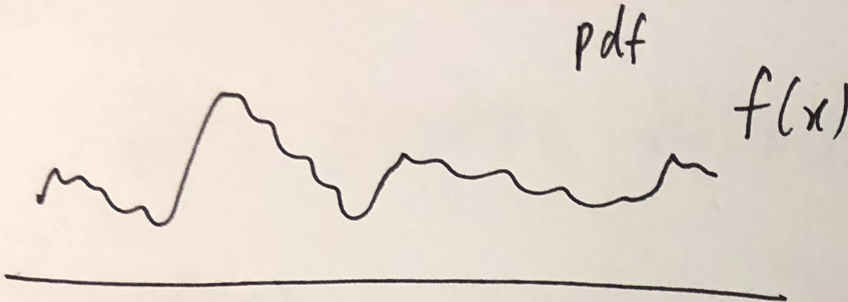
S

$$p(X \in S) = \int_S f(x) \cdot dx$$

cdf: cumulative density function

$$g(a) = p(X \leq a) = \int_{-\infty}^{a} f(x) \cdot dx$$

$$f(x) = \frac{d\, g(x)}{dx}$$



$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx \quad \Big| \quad \underbrace{\sum_x x \cdot p(x)}_{\text{discrete}}$$

$$Var[X] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 \cdot f(x) \cdot dx \quad \Big| \quad \sum_x (x-\mu)^2 p(x)$$

$$\sigma(X) = \sqrt{Var(X)}$$

Markov's

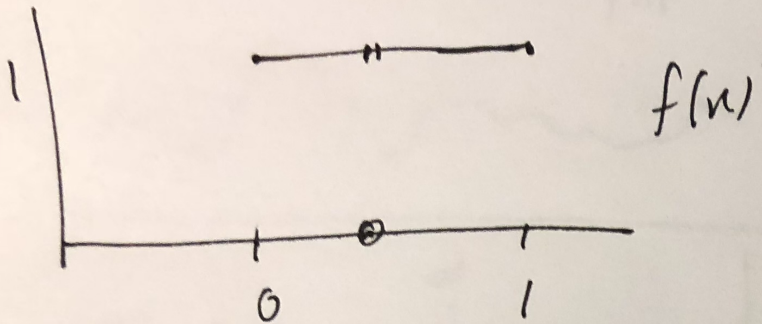
If $X \geq 0$

$$p(X \geq \alpha) \leq \frac{\mathbb{E}[X]}{\alpha}$$

Chebyshev

$\alpha \geq 0$

$$p(|X - \mathbb{E}[X]| \geq \alpha) \leq \frac{var(X)}{\alpha^2}.$$



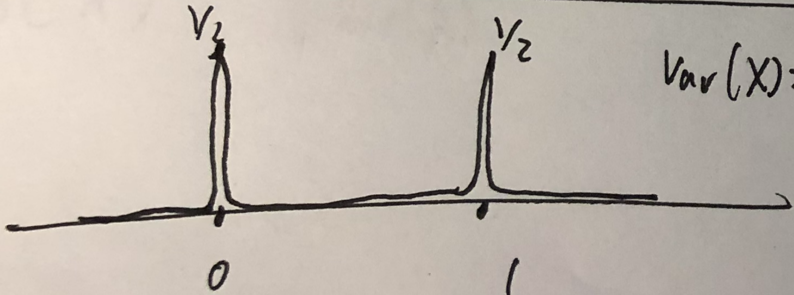
$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx$$

$$= \int_{0}^{1} x \cdot f(x) \cdot dx$$

$$= \int_{0}^{1} x \cdot dx$$

$$= \frac{x^2}{2}\Big|_0^1$$

$$= \frac{1}{2} - 0 = 1/2$$



$Var(X) = \frac{1}{4}$

$Var[X]$

$$= \int_{-\infty}^{\infty} (x - 1/2)^2 \cdot f(x) \cdot dx$$

$$= \int_{0}^{1} (x - 1/2)^2 \, dx$$

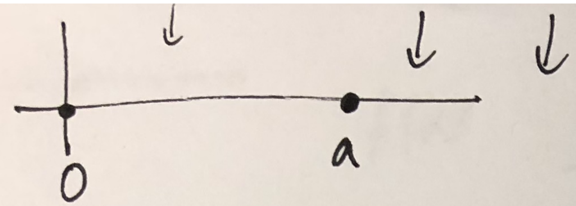
$$= \frac{1}{3}(x - 1/2)^3 \Big|_0^1$$

$$= \frac{(1 - 1/2)^3 - (0 - 1/2)^3}{3} = \frac{1/8 + 1/8}{3}$$

$$= \frac{1}{12}.$$

Chain Rule: $\frac{d\, f(g(x))}{dx} = f'(g(x)) \cdot g'(x)$

P_λ



X : # arrivals in an interval of length τ

$$P_\lambda(X=k) = e^{-\lambda t} \cdot \frac{(\lambda\tau)^k}{k!}$$

Y : time of first arrival.

f : pdf of Y.

g : cdf of Y

$$g(a) = p(Y \le a)$$

$$= 1 - p(Y > a)$$

$$\cancel{= 1 - P_\lambda(0)}$$

$$= 1 - e^{-\lambda a} \frac{\cancel{(\lambda a)^0}}{\cancel{0!}}$$

$$= 1 - e^{-\lambda a}$$

pdf

$$\frac{d(-e^{-\lambda a})}{da} = \lambda \cdot e^{-\lambda a}$$

$$\mathbb{E}[Y] = \frac{1}{\lambda}$$