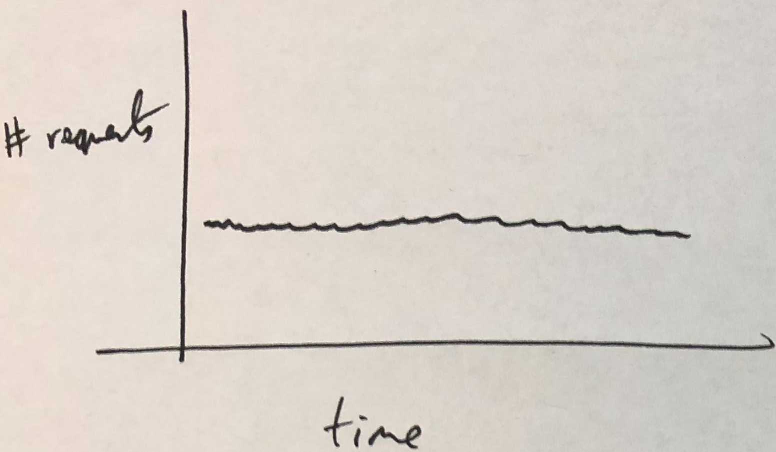
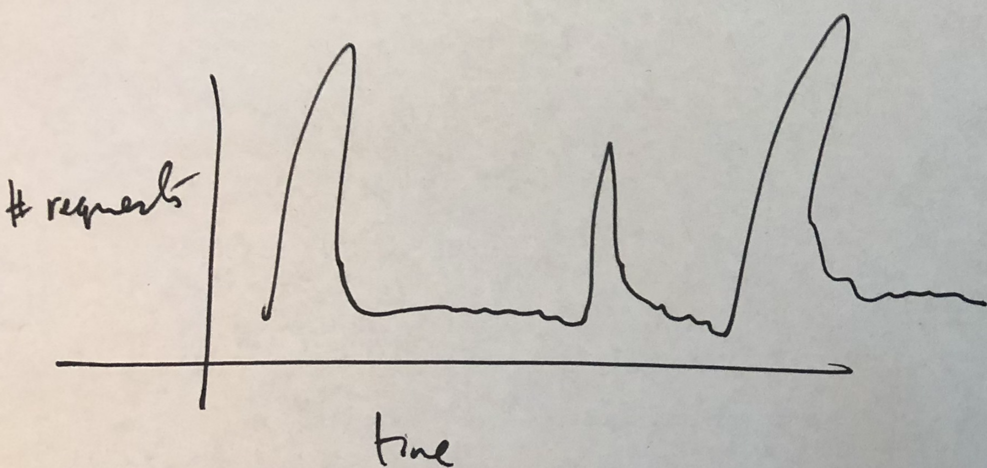
Google

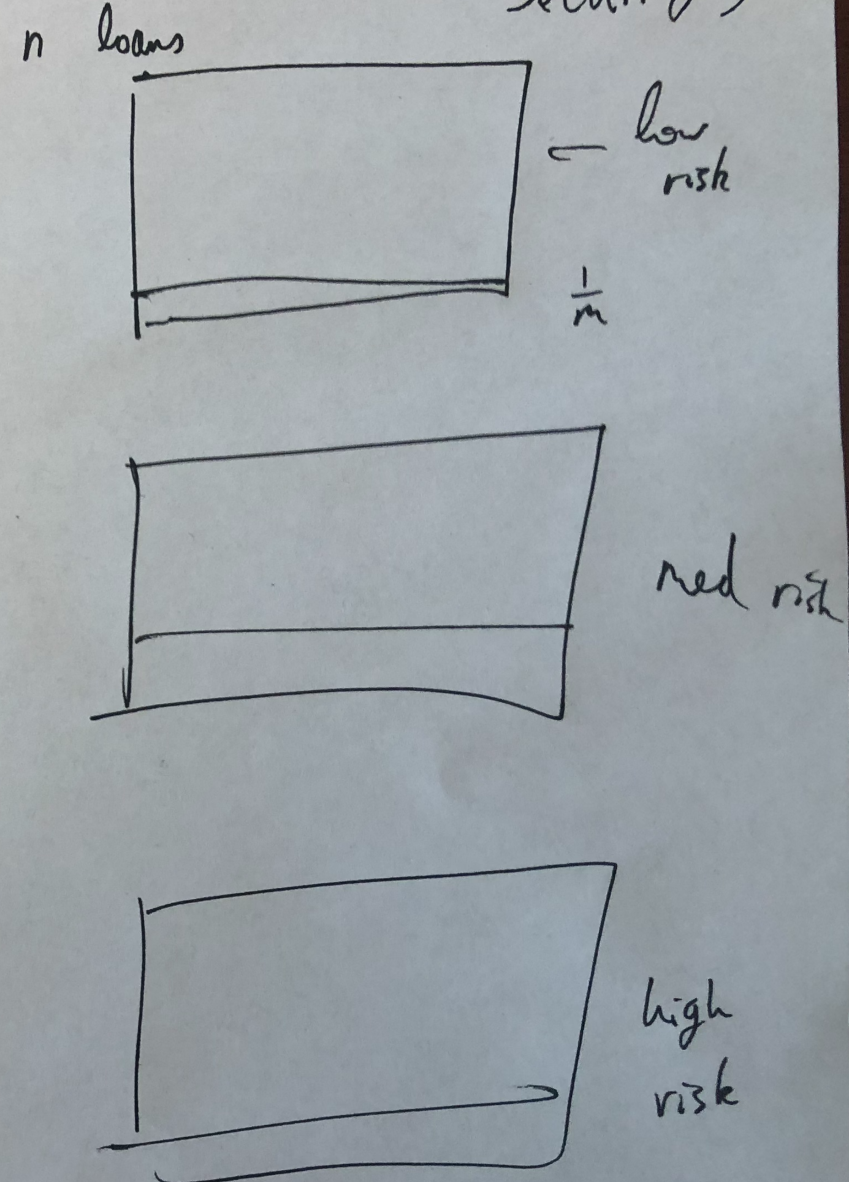




In a given hour: X : # requests)

$$p(X > 10^9)$$

MBS (Mortgage Backed Security)



Poisson Process

parameter : λ

$$P_\lambda(X=k) = e^{-\lambda}\frac{\lambda^k}{k!}$$

$\times \longrightarrow \times$

Bernoulli: Single coin toss with $p(\text{heads}) = p$.

Binomial: n coin tosses with $p(\text{heads}) = p$

X : # heads.

Geometric: First heads in infinite seq. of coin tosses.

Binomial

$$p(X=k) = \binom{n}{k}\cdot p^k (1-p)^{n-k}$$

$\lambda = p.n. \quad \Rightarrow \quad p = \frac{\lambda}{n}$

$$= \cancel{n.(n-1)\ldots(}$$

$$= \frac{n!}{k!\,(n-k)!}\cdot\left(\frac{\lambda}{n}\right)^k\cdot\left(1-\frac{\lambda}{n}\right)^{n-k}$$

$$= \underbrace{\frac{n.(n-1)(n-2)\ldots(n-k+1)}{n\,.\,n\,.\,n\,.\,n}}_{\to 1}\cdot\underbrace{\frac{\lambda^k}{k!}}\cdot\underbrace{\left(1-\frac{\lambda}{n}\right)^{n-k}}_{e}$$

As $n \to \infty$

Recall: $e^{-1} = \lim_{x\to 0}(1-x)^{1/x}$

Binomial

$$p(X=k) = \binom{n}{k} \cdot p^k (1-p)^{n-k}$$

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$$= \underbrace{\frac{n.(n-1)(n-2)\ldots(n-k+1)}{n \cdot n \cdot n \cdot n}}_{\to 1} \cdot \underbrace{\frac{\lambda^k}{k!}} \cdot \underbrace{\left(1-\frac{\lambda}{n}\right)^{n-k}}_{\cancel{e}}$$

As $n \to \infty$

Recall: $e^{-1} = \lim_{x \to 0} (1-x)^{1/x}$

$$\left(1-\frac{\lambda}{n}\right)^{n-k}$$

$$= \underbrace{\left(1-\frac{\lambda}{n}\right)^{-k}}_{\to 1} \cdot \underbrace{\left(1-\frac{\lambda}{n}\right)^{n}}$$

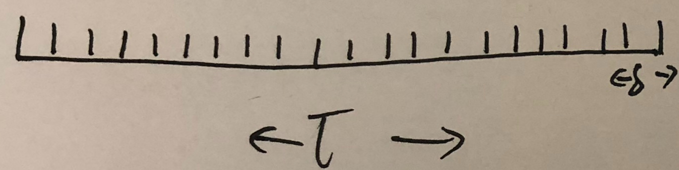
$$= \left(\left(1-\frac{\lambda}{n}\right)^{n/\lambda}\right)^{\lambda}$$

$$\to \left(e^{-1}\right)^{\lambda} = e^{-\lambda}.$$

Say we have interval of length τ.

Expected # of requests is like $\lambda \cdot \tau$.

Model



$\frac{\tau}{\delta}$ coin tosses

each ~~wit~~ giving request with probability.

$$\lambda\tau = \mathbb{E}[\#\text{ requests}] = p \cdot \left(\frac{\tau}{\delta}\right)$$

$$\Rightarrow p = \lambda \cdot \delta.$$

$\Rightarrow$ If you take $\delta \to 0$

$$P_{\lambda\tau}(X=k) = e^{-\lambda\tau} \cdot \frac{(\lambda\tau)^k}{k!}$$

~~$\mathbb{E}[X]$~~

$$P_\lambda(X=k) = e^{-\lambda} \frac{\lambda^k}{k!}$$

$$\mathbb{E}[X] = \lambda = \sum_{k=0}^{\infty} P_\lambda(X=k) \cdot k$$

$$= \sum_{k=0}^{\infty} e^{-\lambda} \cdot \frac{\lambda^k}{k!} \cdot k$$

$$\mathrm{Var}[X] = \lambda.$$

If you take $\delta \to 0$

$$P_{\lambda\tau}(X=k) = e^{-\lambda\tau} \cdot \frac{(\lambda \tau)^k}{k!}$$

$\mathbb{E}[\cancel{X}]$

$$P_\lambda(X=k) = e^{-\lambda} \frac{\lambda^k}{k!}$$

$$\mathbb{E}[X] = \lambda = \sum_{k=0}^{\infty} P_\lambda(X=k) \cdot k$$

$$= \sum_{k=0}^{\infty} e^{-\lambda} \cdot \frac{\lambda^k}{k!} \cdot k$$

$$\mathrm{Var}[X] = \lambda.$$

n trials

$$\mathrm{Var}(X) = n(p-p^2)$$

$$\lambda = np \qquad = np - np^2$$

$$= \lambda - \frac{\lambda^2}{n}$$

$$\mathbb{E}[X^2] = \sum_{k=0}^{\infty} P_\lambda(X=k) \cdot k^2$$

$$\vdots$$

$$= \lambda^2 + \lambda$$

$$\mathrm{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

$$= \lambda^2 + \lambda - (\lambda)^2$$

$$= \lambda.$$