The Discrimination Examples

Study 1: Acceptance rate for men, women is the same

$$p(A|M) = p(A|W)$$

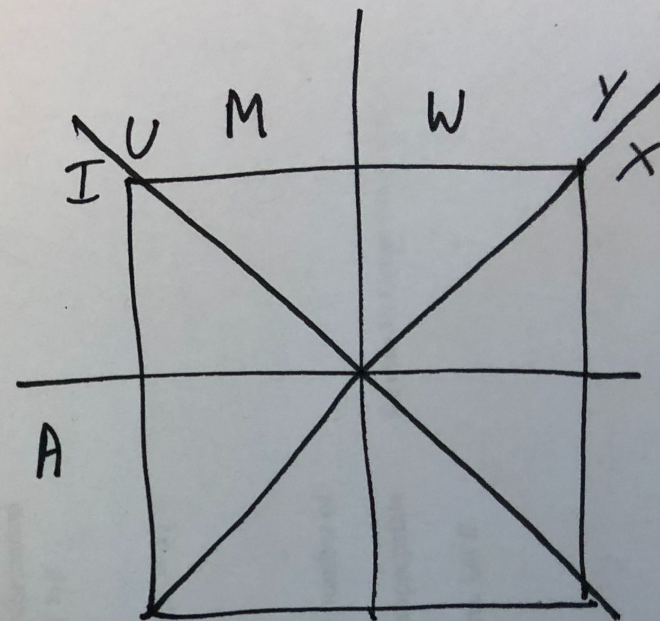
Study 2: In every major, acceptance rate for men is higher

X, Y : majors

$p(A|M,X) > p(A|W,X)$. $p(A|M,Y) > p(A|W,Y)$

Study 3: whether student is int. or not, acceptance rate for women is higher   I: int.   U: U.S.

$p(A|W,I) > p(A|M,I)$, $p(A|W,U) > p(A|M,U)$



<u>FACT:</u> $p(A_1 \cap A_2 \cap A_3)$

$= p(A_1) \cdot p(A_2 | A_1) \cdot p(A_3 | A_1 A_2)$

<u>Ex</u>

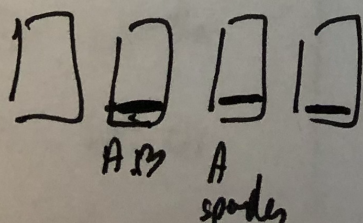
Randomly split a deck of cards (52 cards, 4 aces) into 4 groups of 13 cards each.

What is the prob. that all 4 Aces end up in different groups?

E: 4 Aces in diff grps

$$\frac{|E|}{|\Omega|}$$

$p(A_1)$



A_1: event A of hearts and A of spades are in diff groups.

A_2: A of hearts, spades and diamonds are in diff groups.

A_3: all four aces are in diff groups.

$A_3 = A_1 \cap A_2 \cap A_3$

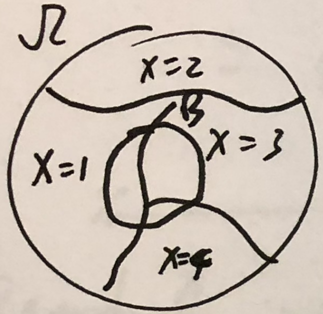
$p(A_3) = p(A_1 \cap A_2 \cap A_3)$

$= p(A_1) \cdot p(A_2 | A_1) \cdot p(A_3 | A_1 A_2)$

$= \frac{39}{51} \cdot \frac{26}{50} \cdot \frac{13}{49}$.

Random variables

Ω



$$X : \Omega \to S = \{H, T\}^{n-1}$$

FACT: $A_1, \ldots, A_n$ disjoint

$$p(B) = p(A_1 \cap B) + p(A_2 \cap B) + \cdots + p(A_n \cap B)$$

FACT: If X is a random variable

$$p(B) = \sum_x p(B \cap X = x)$$

$$= \sum_x p(X = x) . \, p(B | X = x)$$

Ex

n coin tosses

$$p(\text{\# heads is even}) = \frac{1}{2}.$$

X : outcome of fist $n-1$ coin tosses.

E : # heads is even

$$p(E) = \sum_x p(X = x) . \, p(E | X = x)$$

Case 1: # heads in x is even.

$p(E | X = x) = p($last coin tosss being heads$)$

$= 1/2$

Case 2:

...

Ex

n coin tosses

$p(\#\text{ heads is even}) = \frac{1}{2}.$

X : outcome of fist n-1 coin tosses.

E: # heads is even

$$p(E) = \sum_x p(X=x) \cdot p(E|X=x)$$

Case 1: # heads in x is even.

$p(E|X=x) = p(\text{last coin toss being heads}) = 1/2$

Case 2: ...

$$\sum_x p(X=x) \cdot 1/2$$

$$\left(\frac{1}{2}\right) \cdot \left(\sum_x p(X=x)\right)$$

$$= \frac{1}{2}.$$

Indepence

E, F : events

$$p(E \cap F) = p(E) \cdot p(F)$$

$$\Leftrightarrow$$

$$p(E|F) = p(E)$$

$$\Leftrightarrow$$

$$p(F|E) = p(F)$$

X, Y : random variables

For all x, y

$$p(X=x, Y=y) = p(X=x) \cdot p(Y=y)$$

$$\Leftrightarrow$$

$$p(X=x | Y=y) = p(X=x)$$

X, Y, Z are independant

means

$\forall$ For all x, y, z

$$p(X=x \cap Y=y \cap Z=z) = p(X=x) \cdot p(Y=y) \cdot p(Z=z)$$

X —— X