Sets

$S = \{1, 2, 3\}$

$\shortparallel$

$T = \{2, 1, 3\}$

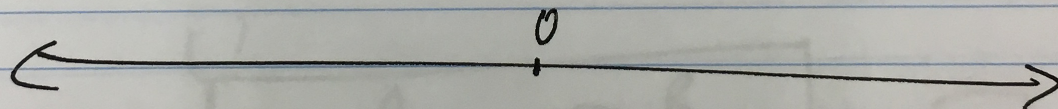
integers $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

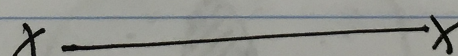
naturals $\mathbb{N} = \{1, 2, \ldots\}$

$[n] = \{1, 2, 3, \ldots, n\}$

$\mathbb{Q} = \{ p/q : p, q \in \mathbb{Z}, q \neq 0 \}$

$\mathbb{R} =$



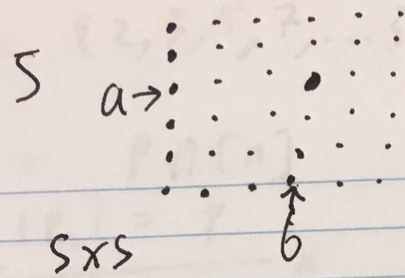


cartesian product

For any set S, $S \times S = \{(a, b) : a, b \in S\}$

$(1, 2) \neq (2, 1)$

$\{1, 2\} = \{2, 1\}$



$\mathbb{R}^2$: real plane

$\mathbb{R}^3$: 3-d space.

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union A, B

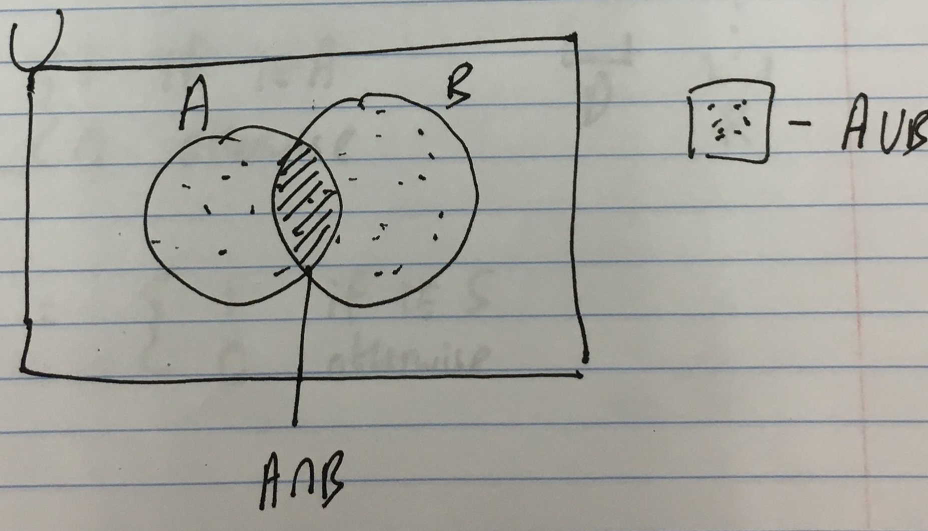
$$A \cup B = \{c : c \in A \text{ or } c \in B\}$$

intersection

$$A \cap B = \{c : c \in A \text{ and } c \in B\}.$$

complement

$$A^c = \{a : a \notin A\}$$



power-set S

2^S : set of all subsets of S

$2^{[n]}$

$P = \{2, 3, 5, 7, \dots\}$

$P_n = P \cap [n]$

$|P_n| = ?$

$x \longrightarrow x$

Sequences

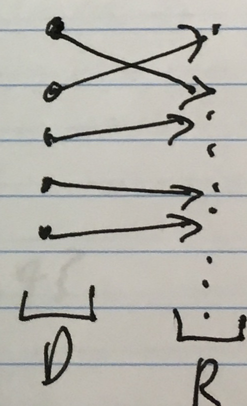
$2, 7, -3, 4, 5$

$1,2,3 \neq 2,1,3$ $\qquad$ $1,3,3,3,3,\dots$

$S \times S$ $\quad$ S^k : k-fold cartesian product

Functions

$f : D \to R$



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set A

indicator fn: $1_A(i) = \begin{cases} 1 & \text{if } i \in A \\ 0 & \text{otherwise} \end{cases}$

If $S \subseteq [n]$

indicator vector: $v_i = \begin{cases} 1 & \text{if } i \in S \\ 0 & \text{otherwise} \end{cases}$

Union - Closed Conjecture

Given $S_1, S_2, \ldots, S_m \subseteq [n]$

are <u>closed under union</u>

if for every i, j, there is a k s.t.

$$S_i \cup S_j = S_k$$

<u>Conj:</u> $m > 1$ If $S_1, \ldots, S_m$ are c. u. u. then there is an $i \in [n]$ which is in $\geq m/2$ sets.

[4]

ϕ, $\{2,3\}$, $\{1,4\}$, $\{1,2,3,4\}$