CSE 312A: Foundations of Computing II
Assignment #6
February 15
due: February 21, at noon.

Instructions:

- **Answers:** When asked for a short answer (such as a single number), also *show and explain your work* briefly. Simplify your final formula algebraically as much as possible, without using your calculator. Then, if the answer is a number rather than a function of some variables, use a calculator to evaluate it and provide the number. For example, for counting problems, your answer might look like this:

  Answer: $\binom{5}{2} - \binom{4}{2} = 4$.

  Explanation: There are $\binom{5}{2}$ ways to select 2 fingers out of the 5, and $\binom{4}{2}$ of them do not involve the thumb.

  Solutions that do not show enough work may not get full credit.

- **Turn-in:** Do not write your name on your pages (your Gradescope account will identify you to us) and do not include a copy of the exercise's question in what you turn in. You must use Gradescope to upload your homework solutions. You will submit a single PDF file containing your solutions to all the exercises in the homework. Each numbered homework question must be answered on its own page (or pages). You must follow the Gradescope prompts that have you link exercise numbers to your pages. You may typeset your solutions on a computer (see here for tutorials and templates) or you can handwrite them, take a picture of (or scan) each handwritten page, and convert the pictures into a single PDF file. You are responsible for making sure that your solution is easily readable, and submitted on time.

1. Your company manufactures computers. The probability that one of your computers is defective is 0.01, independent of other computers made.

   (a) What is the probability that 80 of your computers will contain at least 2 defective computers? Give your answer to 4 significant digits. To begin your solution, state the correct probability distribution with its parameters.

   (b) Approximate the probability of part (a) by using a Poisson distribution, stating the value of its parameter λ. Give your answer to 4 significant digits.

2. The company Moonbound sells luxury spaceships. According to their historical sales, customers buy spaceships at an average rate of 5 per week. Spaceships are expensive for them to stock but, on the other hand, they hate to run out of spaceships and turn away a rich customer. Once a week, Moonbound restocks spaceships for the coming week. They want to have the minimum number of spaceships such that there is less than a 0.1% probability that they have more buying customers than spaceships during the next week. They cannot figure out how many spaceships this should be and turn to you for advice. After asking some questions, you determine that arrivals of paying customers seem to be independent of each other and decide that a Poisson distribution would be a good model. Given this decision, how many spaceships should Moonbound stock for the coming week? Justify you answer.

3. For this exercise, give exact answers as simplified fractions. Compute $\mathrm{E}[X]$ and $\mathrm{Var}(X)$ if X has probability density function given by ...

   (a) $f(x) = \begin{cases} c(1-x^4) & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$.

   Determine the value of c as part of your answer.

   (b) $f(x) = \begin{cases} c/x^3 & \text{if } x > 5 \\ 0 & \text{otherwise} \end{cases}$.

   Determine the value of c as part of your answer.

4. You throw a dart at a circular target of radius r. Your aim is such that the dart is equally likely to hit any point in the target. Let X be the distance of your dart's hit from the center of the target. As a function of r, determine the value m such that $\mathrm{P}(X < m) = \mathrm{P}(X > m)$. Then, for $r = 10$, give the value of m to 3 significant digits.

5. You throw a dart at a circular target of radius r. Let X be the distance of your dart's hit from the center of the target. You have improved and your aim is such that X is an exponential distribution with parameter $4/r$.

   (a) As a function of r, determine the value m such that $\mathrm{P}(X < m) = \mathrm{P}(X > m)$. Then, for $r = 10$, give the value of m to 3 significant digits.

   (b) What is the probability that you miss the target completely? Give your answer to 3 significant digits.

6. A flea of negligible size is trapped in a large, spherical, inflated beach ball with radius r. At this moment, it is equally likely to be at any point within the ball. Let X be the distance of the flea from the center of the ball. For X, find ...

   (a) ... the cumulative distribution function F.

   (b) ... the probability density function f.

   (c) ... the expected value.

   (d) ... the variance.