CSE 312A: Foundations of Computing II
Assignment #4
January 24
due: January 31, at noon.

**Instructions:**

- **Answers:** When asked for a short answer (such as a single number), also *show and explain your work* briefly. Simplify your final formula algebraically as much as possible, without using your calculator. Then, if the answer is a number rather than a function of some variables, use a calculator to evaluate it and provide the number. For example, for counting problems, your answer might look like this:

  Answer: $\binom{5}{2} - \binom{4}{2} = 4$.

  Explanation: There are $\binom{5}{2}$ ways to select 2 fingers out of the 5, and $\binom{4}{2}$ of them do not involve the thumb.

  Solutions that do not show enough work may not get full credit.

- **Turn-in:** Do not write your name on your pages (your Gradescope account will identify you to us) and do not include a copy of the exercise's question in what you turn in. You must use Gradescope to upload your homework solutions. You will submit a single PDF file containing your solutions to all the exercises in the homework. Each numbered homework question must be answered on its own page (or pages). You must follow the Gradescope prompts that have you link exercise numbers to your pages. You may typeset your solutions on a computer (see here for tutorials and templates) or you can handwrite them, take a picture of (or scan) each handwritten page, and convert the pictures into a single PDF file. You are responsible for making sure that your solution is easily readable, and submitted on time.

1. In some games, such as tennis and ping pong, you can reach a state called "deuce". This means that the score is tied and either player wins the game when he or she gets two points ahead of the other player. Suppose you are at deuce. Suppose the probability that you win a single point is $p$ and this is true independently for all points. As a function of $p$, what is the probability that you win the game? You are required to solve this using the following method: from a given score, condition on the outcome of the next point or points, similar to the Gambler's Ruin solution from lecture. Simplify your final answer as much as possible. Then, evaluate it for $p = 3/4$, giving an exact answer as a simplified fraction.

2. A girl with a taste for intricate schemes has 5 lavender and 2 red marbles in her left pocket, and 3 lavender and 4 red marbles in her right pocket. She chooses one of her pockets (called the "chosen pocket") by drawing a randomly chosen marble $D$ from her left pocket: if $D$ is Lavender the Left pocket is the chosen pocket, and if $D$ is Red the Right pocket is the chosen pocket. She returns $D$ to her left pocket. She then draws a randomly chosen marble $M_1$ from the chosen pocket, notes its color, returns it to the same pocket, and then again draws a randomly chosen marble $M_2$ from the same pocket. Give exact answers as simplified fractions.

   (a) What is the probability that $M_1$ is lavender? What is the probability that $M_2$ is lavender?

   (b) If $M_1$ is lavender, what is the probability that $M_2$ is lavender?

(c) Are the events that $M_1$ is lavender and $M_2$ is lavender independent? Justify your answer.

(d) If both $M_1$ and $M_2$ are lavender, what is the probability that $D$ was lavender?

3. As you may remember from basic biology, the human A/B/O blood type system is controlled by one gene for which 3 variants ("alleles") are common in the human population – unsurprisingly called A, B, and O. As with most genes, everyone has 2 copies of this gene, one inherited from the mother and the other from the father, and everyone passes a randomly selected copy to each of their children (probability 1/2 for each copy, independently for each child). Focusing only on A and O, people with AA or AO gene pairs have type A blood; those with OO have type O blood. (A is "dominant", O is "recessive".) Suppose Apple and both of her parents have type A blood, but her sister Olive has type O. Give exact answers as simplified fractions.

   (a) What is the probability that Apple carries an O gene?

   (b) Apple marries a man with type O blood. What is the probability that their first child will have type O blood?

   (c) If their first child had type A blood, what is the probability that Apple carries an O gene?

   (d) If their first child had type A blood, what is the probability that their second child will as well?