CSE 312A: Foundations of Computing II
Assignment #3
January 17
due: January 24, at noon.

Instructions:

- **Answers:** When asked for a short answer (such as a single number), also *show and explain your work* briefly. Simplify your final formula algebraically as much as possible, without using your calculator. Then, if the answer is a number rather than a function of some variables, use a calculator to evaluate it and provide the number. For example, for counting problems, your answer might look like this:

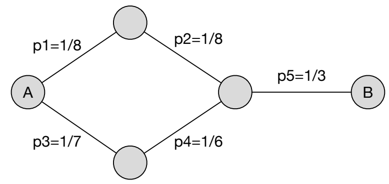
  Answer: $\binom{5}{2} - \binom{4}{2} = 4$.

  Explanation: There are $\binom{5}{2}$ ways to select 2 fingers out of the 5, and $\binom{4}{2}$ of them do not involve the thumb.

  Solutions that do not show enough work may not get full credit.

- **Turn-in:** Do not write your name on your pages (your Gradescope account will identify you to us) and do not include a copy of the exercise's question in what you turn in. You must use Gradescope to upload your homework solutions. You will submit a single PDF file containing your solutions to all the exercises in the homework. Each numbered homework question must be answered on its own page (or pages). You must follow the Gradescope prompts that have you link exercise numbers to your pages. You may typeset your solutions on a computer (see here for tutorials and templates) or you can handwrite them, take a picture of (or scan) each handwritten page, and convert the pictures into a single PDF file. You are responsible for making sure that your solution is easily readable, and submitted on time.

1. A standard deck of cards contains 52 cards. Each card has a value and a suit. There are 13 possible values, and 4 possible suits. The deck is shuffled well and 4 cards are dealt. What is the probability that...
   (a) ...all four cards have the same suit?
   (b) ...all four cards have the same value?
   (c) ...among the four cards, no two have the same value and no two have the same suit?
2. You choose 12 distinct integers between 0 and 2000, inclusive, uniformly at random. What is the probability that among these 12 integers, there is at least one pair whose difference is a multiple of 11? Give an exact answer as a simplified fraction. Hint: Pigeonhole principle and modular arithmetic.
3. In the communications networks diagrammed below, each link has been labeled with the probability that it fails during a given day. Each link fails independently with the given probability. What is the probability that A can communicate with B? That is, the probability that there are one or more paths from A to B all of whose links are working during the day? Give your answer as a function of $p1$, $p2$, $p3$, $p4$, $p5$, and simplify your formula as much as possible. Then, evaluate your formula at the given values.



4. A pharmaceutical company proudly publishes results from a trial of its new test for a certain genetic disorder. The false negative rate is small: the test returns a negative result for only 4% of patients with the disorder. The false positive rate is also small: the test returns a positive result for only 12% of participants that do not have the disorder. Assume that 0.5% (that is, the fraction 0.005) of the population has the disorder. Let's see how good a test this will be and what a test result would mean to you as a patient. Calculate your answers to 2 significant digits.

   (a) What is the probability of having the disorder if you have a negative test result? (Seeing your answer, how reassured should you be if you were the one that had a negative test result?)
   (b) What is the probability of having the disorder if you have a positive test result? (Seeing your answer, how anxious should you be if you were the one that had a positive test result?)
   (c) Repeat part (a) assuming that 15% of the population has the disorder.
   (d) Repeat part (b) assuming that 15% of the population has the disorder.

5. An urn contains 6 red, 8 green, and 9 blue balls. The following is repeated 3 times: a ball is selected from the urn at random and removed (called "sampling *without* replacement"). Give your answers to 3 significant digits.

   (a) What is the probability that all 3 selected balls are the same color?
   (b) What is the probability that all 3 selected balls are different colors?
   (c) Repeat part (a) assuming "sampling *with* replacement". That is, the following is repeated 3 times: a single ball is selected from the urn at random, its color is noted, then it is returned to the urn before the next ball is drawn.
   (d) Repeat part (b) assuming sampling *with* replacement.

6. Rosencrantz and Guildenstern are flipping coins. Guildenstern has a bag with 100 coins in it. All of them are fair coins, except for 8 that each have heads on both sides and 2 that each have tails on both sides. He reaches into his bag without looking, removes a randomly chosen coin, and flips it. Give exact answers expressed as simplified fractions.

   (a) What is the probability that it is one of the 2-headed coins, given that the flip came up heads?
   (b) What is the probability that it is one of the fair coins, given that the flip came up heads?
   (c) What is the probability that it is one of the fair coins, given that the flip came up tails?